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Twist-4 distribution amplitudes of the K^* and ϕ mesons in QCD

Patricia Ball
IPPP, Department of Physics, University of Durham, Durham DH1 3LE, U.K.
E-mail: Patricia.Ball@durham.ac.uk

Vladimir M. Braun and Alexander Lenz
Institut für Theoretische Physik, Universität Regensburg, D–93040 Regensburg, Germany
E-mail: Vladimir.Braun@physik.uni-regensburg.de, Alexander.Lenz@physik.uni-regensburg.de

Abstract: We present a systematic study of twist-4 light-cone distribution amplitudes of the K^* and ϕ meson in QCD. The structure of SU(3)-breaking corrections is studied in detail. Non-perturbative input parameters are estimated from QCD sum rules and a renormalon based model. As a by-product, we give a complete reanalysis of the parameters of the twist-4 ρ-meson distribution amplitudes.

Keywords: Kaon Physics, Phenomenological Models, QCD, Sum Rules.

Contents

1. Introduction

The notion of distribution amplitudes (DAs) [1] refers to matrix elements of nonlocal light-ray operators sandwiched between the hadron state and the vacuum. The physical interpretation of DAs is transparent in the infinite momentum frame [2], in which case DAs correspond to momentum-fraction distributions of partons in a hadron at small transverse separation. Equivalently, DAs can be related to the transverse momentum integrals of the hadron's Bethe-Salpeter wave functions that appear e.g. using the formalism of the light-cone quantization [3]. Schematically,

$$\phi(x) \sim \int^{|k_\perp|<\mu} d^2k_\perp \phi_{\rm BS}(x, k_\perp)\,.$$

The natural application of DAs in phenomenology are exclusive hard processes with large momentum transfer which can be calculated using factorisation methods. Apart from meson/baryon electromagnetic form factors, this includes a large class of phenomenologically very interesting B meson decays, such as weak decay form factors [4], the non-leptonic decays $B \to M_1 M_2$, where M_i is a light meson [5], and also rare radiative and semileptonic decays, $B \to M\gamma$ [6] and $B \to M\ell^+\ell^-$ [7], which involve flavour-changing neutral currents and are heavily suppressed in the Standard Model, but sensitive to new-physics effects. For these processes, the shape of the DAs is very important: in $B \to \rho\gamma$ vs. $B \to K^*\gamma$, for example, the size of SU(3) breaking in the relevant DAs is presently the dominant source of theoretical uncertainty [8]. All these decays will be studied in detail at the forthcoming LHC, which makes the detailed investigation of the various leading and higher-twist DAs both timely and relevant.

The crucial point and main technical difficulty in the construction of higher-twist DAs is the necessity to satisfy the exact equations of motion (EOM), which yield relations between physical effects of different origin: for example, using EOM, the contributions of orbital angular momentum in the valence component of the wave function can be expressed (for mesons) in terms of contributions of higher Fock states. An appropriate framework for implementing these constraints was developed in ref. [9]: it is based on the derivation of EOM relations for non-local light-ray operators [10], which are solved order by order in the conformal expansion; see ref. [11] for a review and further references. In this way it is possible to construct self-consistent approximations for the DAs, which involve a minimum number of hadronic parameters. Another approach, based on the study of renormalons, was suggested for twist 4 in refs. [12, 13]: this technique is appealing as it allows one to obtain an estimate of high-order contributions to the conformal expansion which are usually omitted. In ref. [14] we have generalized this approach to include SU(3)-breaking corrections and have shown how to combine renormalon-based estimates of "genuine" twist-4 effects with meson mass corrections.

In a series of previous papers, refs. [15, 16], we have developed the corresponding formalism for vector mesons. In ref. [17] the analysis of twist-3 DAs of light vector mesons, ρ, K^* and ϕ, was completed, including all SU(3)-breaking effects. In this paper, we put together the last missing pieces, completing the study of the corresponding DAs of twist 4, with the main emphasis on the calculation of SU(3)-breaking effects in the relevant hadronic matrix elements. These corrections come from two different sources:

- SU(3) breaking of hadronic parameters: these effects are known for twist-2 and -3 parameters, see refs. [17, 18], but have not been studied for twist-4 parameters before. It is convenient to separate the effects that are parametrically of order $m_s - m_q$ and vanish in the limit of equal quark mass, i.e. for ρ and ϕ, and those of order $m_s + m_q$. In a slight abuse of language we will refer to the former as G-parity-breaking, and the latter as G-parity-conserving, respectively. The G-parity-breaking effects appear to be more involved. For twist-2 DAs, they have been calculated, to lowest order in the conformal expansion in refs. [15, 18−21], and for twist-3 DAs in ref. [17]; they are unknown for twist-4 DAs;

- explicit quark-mass corrections: these affect only higher-twist DAs and are induced by terms in $m_s \pm m_q$ that appear in the QCD EOM which relate twist-4 DAs to each other and to twist-2 and -3 DAs and in the renormalization group equations. The mass corrections to vector meson DAs have been calculated to twist-4 accuracy in ref. [16]. Quark-mass corrections to the evolution of DAs under a change of the renormalisation scale have so far only been calculated for twist-3 DAs [17].

We shall study all these effects in this paper. The corresponding analysis of light pseudoscalar meson DAs, π and K, can be found in refs. [9, 14, 22].

In addition, in this work we present a new analysis of the parameters of the twist-4 DAs of the ρ meson. This update is long overdue: the "standard" values for these parameters can be traced back to a nearly 20-years-old work, ref. [23] (see also [16]), and are in fact crude estimates obtained by dividing the leading QCD contribution to the relevant correlation functions by the typical hadronic scale. In this work we present, for the first time, a complete treatment of the twist-4 matrix elements within the QCD sum rule method. In particular, we resolve a puzzling discrepancy between the estimate of a certain next-to-leading order (NLO) parameter, in conformal spin, in ref. [16] and in the renormalon model [13].

The presentation is organised as follows: in section 2 we introduce notations and shortly review twist-2 and -3 DAs. In section 3 we introduce the complete set of chiral-even and in section 4 chiral-odd twist-4 DAs. The conformal expansions of all DAs are worked out to NLO accuracy in conformal spin and reduced to a minimum number of non-perturbative hadronic parameters by solving the EOM constraints. In section 5, we present models for these DAs, based on the calculation of the hadronic parameters from QCD sum rules. We summarise and conclude in section 6. Details of the QCD sum rule calculations are given in the appendices.

2. General framework

2.1 Kinematics and notations

Light-cone meson DAs are defined in terms of matrix elements of non-local light-ray operators extended along a certain light-like direction z_μ, $z^2 = 0$, and sandwiched between the vacuum and the meson state. Following ref. [14], we adopt the generic notations

$$\phi^\lambda_{t;M}(u),\ \psi^\lambda_{t;M}(u),\ \ldots \tag{2.1}$$

and

$$\Phi^\lambda_{t;M}(\underline{\alpha}),\ \Psi^\lambda_{t;M}(\underline{\alpha}),\ \ldots \tag{2.2}$$

for two- and three-particle DAs, respectively. The superscript λ denotes the polarisation of the vector meson: $\lambda = \parallel$ ($\perp$) for longitudinal (transverse) polarisation. The first subscript $t = 2, 3, 4$ stands for the twist; the second, $M = \rho, K^*, \ldots$, specifies the meson. For definiteness, we will write most expressions for K^* mesons, i.e. $s\bar{q}$ bound states with $q = u, d$. Whenever relevant, we will include quark-mass corrections in the form $m_s \pm m_q$, which allows one to obtain the results for ϕ mesons by $m_q \to m_s$. The variable u in the definition of two-particle DAs always refers to the momentum fraction carried by the quark, $u = u_s$, whereas $\bar{u} \equiv 1 - u = u_{\bar{q}}$ is the antiquark momentum fraction. The set of variables in the three-particle DAs, $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\} = \{\alpha_s, \alpha_{\bar{q}}, \alpha_g\}$, corresponds to the momentum fractions carried by the quark, antiquark and gluon, respectively.

To facilitate the light-cone expansion, it is convenient to use light-like vectors p_μ and z_μ instead of the meson's four-momentum P_μ and the coordinate x_μ:

$$
\begin{aligned}
z_\mu &= x_\mu - P_\mu \frac{1}{m_{K^*}^2}\left[xP - \sqrt{(xP)^2 - x^2 m_{K^*}^2}\right] = x_\mu\left[1 - \frac{x^2 m_{K^*}^2}{4(zp)^2}\right] - \frac{1}{2} p_\mu \frac{x^2}{zp} + \mathrm{O}(x^4)\,,\\
p_\mu &= P_\mu - \frac{1}{2}\, z_\mu \frac{m_{K^*}^2}{pz}\,.
\end{aligned}
\tag{2.3}
$$

The meson's polarization vector $e^{(\lambda)}$ can be decomposed into projections onto the two light-like vectors and the orthogonal plane as follows:

$$
e_\mu^{(\lambda)} = \frac{e^{(\lambda)}z}{pz}\, p_\mu + \frac{e^{(\lambda)}p}{pz}\, z_\mu + e_{\perp\mu}^{(\lambda)} = \frac{e^{(\lambda)}z}{pz}\left(p_\mu - \frac{m_{K^*}^2}{2pz}\, z_\mu\right) + e_{\perp\mu}^{(\lambda)}\,. \tag{2.4}
$$

We also need the projector $g_{\mu\nu}^{\perp}$ onto the directions orthogonal to p and z,

$$
g_{\mu\nu}^{\perp} = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu)\,, \tag{2.5}
$$

and will often use the notations

$$
a_z \equiv a_\mu z^\mu, \qquad b_p \equiv b_\mu p^\mu \tag{2.6}
$$

for arbitrary four-vectors a_μ and b_μ.

The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$. Our convention for the covariant derivative is $D_\mu = \partial_\mu - igA_\mu$. Sometimes, a different convention for the sign of g is used in the literature: $D_\mu = \partial_\mu + igA_\mu$. The sign of g is relevant for the parameters of three-particle DAs.

2.2 Conformal expansion

A convenient tool to study DAs is provided by conformal expansion, see ref. [11] for a review. The underlying idea is similar to partial-wave decomposition in quantum mechanics and allows one to separate transverse and longitudinal variables in the Bethe-Salpeter wave-function. The dependence on transverse coordinates is formulated as scale dependence of the relevant operators and is governed by renormalisation-group equations, the dependence on the longitudinal momentum fractions is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group SL(2,$\mathbb{R}$). The main rationale behind using the conformal expansion in the present context is that the EOM always relate contributions of the same spin; a truncation of the conformal expansion to a certain order is, therefore, consistent with the EOM.

To construct the conformal expansion for an arbitrary multi-particle distribution, one first has to decompose each constituent field into components with fixed Lorentz-spin projection onto the light-cone. Each such component has conformal spin

$$
j = \frac{1}{2}\,(l + s),
$$

where l is the canonical dimension and s the (Lorentz-) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. A quark field is decomposed as $\psi_+ \equiv \Lambda_+\psi$ and $\psi_- = \Lambda_-\psi$ with spin projection operators $\Lambda_+ = \gamma_p\gamma_z/(2pz)$ and $\Lambda_- = \gamma_z\gamma_p/(2pz)$, corresponding to $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $G_{z\perp}$ corresponds to $s = +1$, $G_{p\perp}$ to $s = -1$, and both $G_{\perp\perp}$ and G_{zp} correspond to $s = 0$. Multi-particle states built of fields with definite Lorentz-spin projection can be expanded in irreducible representations of SL(2,$\mathbb{R}$) with increasing conformal spin. The explicit expression for the DA of an m-particle state with the lowest possible conformal spin $j = j_1 + \ldots + j_m$, the so-called asymptotic DA, is given by [9]

$$\phi_{as}(\alpha_1, \alpha_2, \cdots, \alpha_m) = \frac{\Gamma(2j_1 + \cdots + 2j_m)}{\Gamma(2j_1)\cdots\Gamma(2j_m)}\, \alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}. \tag{2.7}$$

Multi-particle irreducible representations with higher spin $j + n, n = 1, 2, \ldots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$), which are orthogonal over the weight function (2.7). For the two-particle DAs these are Gegenbauer polynomials, a convenient basis of orthogonal conformal polynomials for the three-particle DAs is given in appendix A of ref. [11] (see also [24]).

The anomalous dimensions of higher conformal amplitudes do, generally, increase logarithmically with the conformal spin, but a complete analysis of twist-4 anomalous dimensions is still lacking. It follows that all DAs approach their asymptotic form in the asymptotic limit $\alpha_s \to 0$, i.e. at the scale $\mu \to \infty$. For practical applications one usually assumes that the conformal expansion is converging fast enough, so that a truncation after the few first terms is sufficient. The renormalon model of refs. [12, 13] presents an attempt to test this assumption, by giving an upper bound for possible higher-spin contributions.

2.3 Twist-2 distributions

The twist-2 DAs $\phi^{\parallel,\perp}_{2;K^*}$ of K^* mesons are defined in terms of the following matrix elements of non-local operators ($\xi = 2u - 1$) [15]:

$$\begin{aligned}\langle 0|\bar q(x)\gamma_\mu s(-x)|K^*(P,\lambda)\rangle = f^\parallel_{K^*} m_{K^*} \Bigg\{ &\frac{e^{(\lambda)}x}{Px}\, P_\mu \int_0^1 du\, e^{i\xi Px} \left[\phi^\parallel_{2;K^*}(u) + \frac{1}{4}\, m^2_{K^*} x^2 \phi^\parallel_{4;K^*}(u)\right] \\ &+ \left(e^{(\lambda)}_\mu - P_\mu \frac{e^{(\lambda)}x}{Px}\right) \int_0^1 du\, e^{i\xi Px}\, \phi^\perp_{3;K^*}(u) \\ &- \frac{1}{2}\, x_\mu \frac{e^{(\lambda)}x}{(Px)^2}\, m^2_{K^*} \int_0^1 du\, e^{i\xi Px} \left[\psi^\parallel_{4;K^*}(u) + \phi^\parallel_{2;K^*}(u) - 2\phi^\perp_{3;K^*}(u)\right] + \ldots \Bigg\},\end{aligned} \tag{2.8}$$

$$\begin{aligned}&\langle 0|\bar q(x)\sigma_{\mu\nu} s(-x)|K^*(P,\lambda)\rangle = \\ &\quad if^\perp_{K^*} \Bigg\{ (e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu) \int_0^1 du\, e^{i\xi Px} \left[\phi^\perp_{2;K^*}(u) + \frac{1}{4}\, m^2_{K^*} x^2 \phi^\perp_{4;K^*}(u)\right] \\ &\quad + (P_\mu x_\nu - P_\nu x_\mu)\, \frac{e^{(\lambda)}x}{(Px)^2}\, m^2_{K^*} \int_0^1 du\, e^{i\xi Px} \left[\phi^\parallel_{3;K^*}(u) - \frac{1}{2}\phi^\perp_{2;K^*}(u) - \frac{1}{2}\psi^\perp_{4;K^*}(u)\right] \\ &\quad + \frac{1}{2}\, (e^{(\lambda)}_\mu x_\nu - e^{(\lambda)}_\nu x_\mu)\, \frac{m^2_{K^*}}{Px} \int_0^1 du\, e^{i\xi Px} \left[\psi^\perp_{4;K^*}(u) - \phi^\perp_{2;K^*}(u)\right] + \ldots \Bigg\}.\end{aligned} \tag{2.9}$$

The above relations also include twist-3 and -4 two-particle DAs. The dots stand for further terms in x^2 which are of twist 5 or higher. The normalisation of all these DAs is given by

$$\int_0^1 du\,\phi(u) = 1\,. \tag{2.10}$$

The conformal expansion goes in terms of Gegenbauer polynomials:

$$\phi_2^{\parallel,\perp}(u,\mu) = 6u\bar{u}\left\{1 + \sum_{n=1}^{\infty} a_n^{\parallel,\perp}(\mu) C_n^{3/2}(2u-1)\right\}. \tag{2.11}$$

In this paper, we include terms up to NLO in conformal spin, i.e. truncate after $n = 2$. The dependence of the Gegenbauer moments a_n on the renormalisation-scale μ has been reviewed in ref. [17], together with the numerical values of a_n and the decay constants $f_V^{\parallel,\perp}$; these values are given in section 5.

2.4 Twist-3 distributions

To twist-3 accuracy, there is a total of four two-particle DAs and three three-particle DAs. Two of the former, $\phi_{3;K^*}^{\perp}$ and $\phi_{3;K^*}^{\parallel}$, have already been defined in the previous subsection. The other two are given by

$$\begin{aligned}
\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 s(-z)|K^*(P,\lambda)\rangle &= \frac{1}{2} f_{K^*}^{\parallel} m_{K^*} \epsilon_\mu{}^{\nu\alpha\beta} e_\nu^{(\lambda)} p_\alpha z_\beta \int_0^1 du\, e^{i\xi px} \psi_{3;K^*}^{\perp}(u)\,, && (2.12)\\
\langle 0|\bar{q}(z) s(-z)|K^*(P,\lambda)\rangle &= -if_{K^*}^{\perp}(e^{(\lambda)}z) m_{K^*}^2 \int_0^1 du\, e^{i\xi pz} \psi_{3;K^*}^{\parallel}(u) && (2.13)
\end{aligned}$$

with the normalisation

$$\int_0^1 du\,\psi_{3;K^*}^{\parallel(\perp)}(u) = 1 - \frac{f_{K^*}^{\parallel(\perp)}}{f_{K^*}^{\perp(\parallel)}} \frac{m_s + m_q}{m_{K^*}}\,. \tag{2.14}$$

The three-particle DAs are given by:

$$\begin{aligned}
\langle 0|\bar{q}(z) g\widetilde{G}_{\beta z}(vz)\gamma_z\gamma_5 s(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^{\parallel} m_{K^*} (pz)^2 e_{\perp\beta}^{(\lambda)} \widetilde{\Phi}_{3;K^*}^{\parallel}(v,pz) + \ldots\,,\\
\langle 0|\bar{q}(z) gG_{\beta z}(vz) i\gamma_z s(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^{\parallel} m_{K^*} (pz)^2 e_{\perp\beta}^{(\lambda)} \Phi_{3;K^*}^{\parallel}(v,pz) + \ldots\,,\\
\langle 0|\bar{q}(z) gG_{z\beta}(vz)\sigma_{z\beta} s(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^{\perp} m_{K^*}^2 (e^{(\lambda)}z)(pz) \Phi_{3;K^*}^{\perp}(v,pz)\,, && (2.15)
\end{aligned}$$

where the dots denote terms of higher twist and we use the short-hand notation

$$\mathcal{F}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2-\alpha_1+v\alpha_3)} \mathcal{F}(\underline{\alpha}) \tag{2.16}$$

with $\mathcal{F}(\underline{\alpha})$ a three-particle DA. $\underline{\alpha}$ is the set of parton momentum fractions $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$ and the integration measure $\mathcal{D}\underline{\alpha}$ is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\, \delta\left(1 - \sum \alpha_i\right). \tag{2.17}$$

The above DAs are not independent of each other, and their mutual interrelations have been unravelled in ref. [15], including quark-mass corrections. Explicit expressions for the conformal expansion are given in ref. [17], together with the μ-dependence of hadronic parameters. Numerical values are given in section 5.

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3. Chiral-even twist-4 distributions

In this section we derive expressions for the chiral-even two- and three-particle twist-4 DAs of the K^* to NLO in the conformal expansion. The corresponding expressions for the ρ were obtained in ref. [16]. In this paper we include also G-parity-violating and explicit quark-mass corrections. We compare the resulting DAs obtained in conformal expansion with those following from the renormalon model developed in ref. [13]. Numerical estimates of the hadronic input parameters and the resulting DAs are discussed in section 5.

We start with the three-particle distributions. The analysis closely follows that of ref. [16]. There are four chiral-even K^* three-particle DAs of twist 4, defined as [16]:[1]

$$
\begin{aligned}
\langle 0|\bar q(z)\gamma_\mu\gamma_5 g\widetilde G_{\alpha\beta}(vz)s(-z)|K^*(P,\lambda)\rangle = {}& p_\mu\left(e^{(\lambda)}_{\perp\alpha}p_\beta - e^{(\lambda)}_{\perp\beta}p_\alpha\right) f^{\parallel}_{K^*} m_{K^*}\widetilde\Phi^{\parallel}_{3;K^*}(v,pz)\\
&+\left(p_\alpha g^\perp_{\beta\mu} - p_\beta g^\perp_{\alpha\mu}\right)\frac{e^{(\lambda)}z}{pz} f^{\parallel}_{K^*} m^3_{K^*}\widetilde\Phi^{\parallel}_{4;K^*}(v,pz)\\
&+p_\mu\left(p_\alpha z_\beta - p_\beta z_\alpha\right)\frac{e^{(\lambda)}z}{(pz)^2} f^{\parallel}_{K^*} m^3_{K^*}\widetilde\Psi^{\parallel}_{4;K^*}(v,pz)+\ldots,
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
\langle 0|\bar q(z)i\gamma_\mu g G_{\alpha\beta}(vz)s(-z)|K^*(P,\lambda)\rangle = {}& p_\mu\left(e^{(\lambda)}_{\perp\alpha}p_\beta - e^{(\lambda)}_{\perp\beta}p_\alpha\right) f^{\parallel}_{K^*} m_{K^*}\Phi^{\parallel}_{3;K^*}(v,pz)\\
&+\left(p_\alpha g^\perp_{\beta\mu} - p_\beta g^\perp_{\alpha\mu}\right)\frac{e^{(\lambda)}z}{pz} f^{\parallel}_{K^*} m^3_{K^*}\Phi^{\parallel}_{4;K^*}(v,pz)\\
&+p_\mu\left(p_\alpha z_\beta - p_\beta z_\alpha\right)\frac{e^{(\lambda)}z}{(pz)^2} f^{\parallel}_{K^*} m^3_{K^*}\Psi^{\parallel}_{4;K^*}(v,pz)+\ldots\,;
\end{aligned}
\tag{3.2}
$$

the dots denote terms of twist 5 and higher.

$\Psi^{\parallel}_{4;K^*}$ and $\widetilde\Psi^{\parallel}_{4;K^*}$ correspond to the light-cone projection $\gamma_z G_{zp}$ which picks up the $s=+1/2$ components of both quark and antiquark field and the $s=0$ component of the gluon field. The conformal expansion then reads:

$$
\begin{aligned}
\Psi^{\parallel}_{4;K^*}(\underline\alpha) &= 120\alpha_1\alpha_2\alpha_3[\psi^{\parallel}_0 + \psi^{\parallel}_1(\alpha_1-\alpha_2) + \psi^{\parallel}_2(3\alpha_3-1)+\ldots],\\
\widetilde\Psi^{\parallel}_{4;K^*}(\underline\alpha) &= 120\alpha_1\alpha_2\alpha_3[\widetilde\psi^{\parallel}_0 + \widetilde\psi^{\parallel}_1(\alpha_1-\alpha_2) + \widetilde\psi^{\parallel}_2(3\alpha_3-1)+\ldots].
\end{aligned}
\tag{3.3}
$$

G-parity implies that, for the ρ and ϕ meson, $\psi^{\parallel}_0 = \psi^{\parallel}_2 = \widetilde\psi^{\parallel}_1 = 0$, whereas for the K^* meson $\psi^{\parallel}_0$, $\psi^{\parallel}_2$ and $\widetilde\psi^{\parallel}_1$ are $\mathcal{O}(m_s-m_q)$.

In turn, the DAs $\Phi_{4;K^*}$ and $\widetilde\Phi_{4;K^*}$ correspond to the light-cone projection $\gamma_\perp G_{z\perp}$, which is a mixture of different quark-spin states with $s_q=+1/2, s_{\bar q}=-1/2$ and $s_q=-1/2, s_{\bar q}=+1/2$, respectively. In both cases $s=+1$ for the gluon. We separate the different quark-spin projections by introducing the auxiliary amplitudes $\Phi^{\uparrow\downarrow}$ and $\Phi^{\downarrow\uparrow}$, defined as

$$
\begin{aligned}
\langle 0|\bar q(z)g\widetilde G_{\mu\nu}(vz)\gamma_z\gamma_\alpha\gamma_5\gamma_p s(-z)|K^*(P,\lambda)\rangle &= f^{\parallel}_{K^*}m^3_{K^*}(e^{(\lambda)}z)(p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu})\Phi^{\uparrow\downarrow}(v,pz)\,,\\
\langle 0|\bar q(z)g\widetilde G_{\mu\nu}(vz)\gamma_p\gamma_\alpha\gamma_5\gamma_z s(-z)|K^*(P,\lambda)\rangle &= f^{\parallel}_{K^*}m^3_{K^*}(e^{(\lambda)}z)(p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu})\Phi^{\downarrow\uparrow}(v,pz)\,.
\end{aligned}
\tag{3.4}
$$

The distributions $\Phi^{\parallel}_{4;K^*}$ and $\widetilde\Phi^{\parallel}_{4;K^*}$ are then given by

$$
\widetilde\Phi_{4;K^*}(\underline\alpha) = \frac{1}{2}\Big[\Phi^{\uparrow\downarrow}(\underline\alpha)+\Phi^{\downarrow\uparrow}(\underline\alpha)\Big],\quad \Phi_{4;K^*}(\underline\alpha) = \frac{1}{2}\Big[\Phi^{\uparrow\downarrow}(\underline\alpha)-\Phi^{\downarrow\uparrow}(\underline\alpha)\Big]. \tag{3.5}
$$

[1] In the notation of ref. [16], $\Phi^{\parallel}_{4;K^*}=\Phi$, $\Psi^{\parallel}_{4;K^*}=\Psi$, $\widetilde\Phi^{\parallel}_{4;K^*}=\widetilde\Phi$, $\widetilde\Psi^{\parallel}_{4;K^*}=\widetilde\Psi$.

$\Phi^{\uparrow\downarrow}$ and $\Phi^{\downarrow\uparrow}$ have a regular expansion in terms of conformal polynomials:

$$\begin{aligned}\Phi^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_2\alpha_3^2\left[\phi_0^{\uparrow\downarrow} + \phi_1^{\uparrow\downarrow}(\alpha_3 - 3\alpha_1) + \phi_2^{\uparrow\downarrow}\left(\alpha_3 - \frac{3}{2}\alpha_2\right)\right],\\ \Phi^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_1\alpha_3^2\left[\phi_0^{\downarrow\uparrow} + \phi_1^{\downarrow\uparrow}(\alpha_3 - 3\alpha_2) + \phi_2^{\downarrow\uparrow}\left(\alpha_3 - \frac{3}{2}\alpha_1\right)\right].\end{aligned} \tag{3.6}$$

For the ρ and ϕ meson, G-parity implies

$$\Phi^{\uparrow\downarrow}_{4;\rho(\phi)}(\alpha_1,\alpha_2) = \Phi^{\downarrow\uparrow}_{4;\rho(\phi)}(\alpha_2,\alpha_1)\,, \tag{3.7}$$

so that $\phi_i^{\uparrow\downarrow} \equiv \phi_i^{\downarrow\uparrow}$.[2] For K^*, we write

$$\phi_i^{\uparrow\downarrow} = \phi_i^{\parallel} + \theta_i^{\parallel}, \qquad \phi_i^{\downarrow\uparrow} = \phi_i^{\parallel} - \theta_i^{\parallel}, \tag{3.8}$$

where the $\theta_i^{\parallel}$ are the G-parity-violating corrections. Using (3.5), we readily derive the following expressions:

$$\begin{aligned}\widetilde{\Phi}^{\parallel}_{4;K^*}(\underline{\alpha}) &= 30\alpha_3^2\Big\{\phi_0^{\parallel}(1-\alpha_3) + \phi_1^{\parallel}\left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right]\\ &\quad + \phi_2^{\parallel}\left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\theta_0^{\parallel} + \alpha_3\theta_1^{\parallel} + \frac{1}{2}(5\alpha_3-3)\theta_2^{\parallel}\right]\Big\},\\ \Phi^{\parallel}_{4;K^*}(\underline{\alpha}) &= 30\alpha_3^2\Big\{\theta_0^{\parallel}(1-\alpha_3) + \theta_1^{\parallel}\left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right]\\ &\quad + \theta_2^{\parallel}\left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\phi_0^{\parallel} + \alpha_3\phi_1^{\parallel} + \frac{1}{2}(5\alpha_3-3)\phi_2^{\parallel}\right]\Big\}.\end{aligned} \tag{3.9}$$

In addition, we introduce one more three-particle DA $\Xi^{\parallel}_{4;K^*}(\underline{\alpha})$ [13]:

$$\langle 0\,|\bar{q}(z)\gamma_\alpha g D_\mu G_{\mu\nu}(vz)s(-z)|\,K^*(P,\lambda)\rangle = f^{\parallel}_{K^*}m^3_{K^*}p_\alpha p_\nu\,\frac{e^{(\lambda)}z}{pz}\,\Xi^{\parallel}_{4;K^*}(v,pz) + \ldots \tag{3.10}$$

The Lorentz structure $p_\alpha p_\nu$ is the only one relevant at twist 4. Because of the EOM $D^\alpha G^A_{\alpha\beta} = -g\sum_q \bar{q}t^A\gamma_\beta q$, where the summation goes over all light flavors, $\Xi^{\parallel}_{4;K^*}$ can be viewed as describing either a quark-antiquark-gluon or a specific four-quark Fock-state of the K^*, with the quark-antiquark pair in a colour-octet state and at the same space-time point. The conformal expansion of $\Xi^{\parallel}_{4;K^*}$ starts with the conformal spin $j=4$ and reads

$$\Xi^{\parallel}_{4;K^*}(\underline{\alpha}) = 840\alpha_1\alpha_2\alpha_3^3\left[\xi_0^{\parallel} + \ldots\right], \tag{3.11}$$

where $\xi_0^{\parallel}$ is dimensionless. The dots stand for terms with higher conformal spin $j = 5, 6, \ldots$, which are beyond our accuracy. $\Xi_4^{\parallel}$ was not considered in ref. [16] because $\xi_0^{\parallel}$ is G-odd and the first G-even term only occurs at the next order in the conformal expansion, for $j=5$.

Eqs. (3.3), (3.9) and (3.11) represent the most general parametrization of the chiral-even twist-4 DAs to NLO in the conformal-spin expansion and involve 13 non-perturbative parameters. Not all of them are independent, though. In the following, we shall

[2]This implies, in particular, that only one of the DAs $\Phi_{4;\rho(\phi)}$ and $\widetilde{\Phi}_{4;\rho(\phi)}$ is dynamically independent.

establish their mutual relations and also express all leading order (LO) and the G-even NLO expansion coefficients in terms of matrix elements of local operators.

Except for $\Xi^{\parallel}_{4;K^*}$, the asymptotic three-particle DAs correspond to contributions of the lowest conformal spin $j = j_s + j_{\bar q} + j_g = 3$. The parameters $\psi^{\parallel}_0$, $\widetilde\psi^{\parallel}_0$, $\phi^{\parallel}_0$ and $\theta^{\parallel}_0$ multiplying the asymptotic DAs can be expressed in terms of local matrix elements as

$$\begin{aligned}\langle 0|\bar q g\widetilde G_{\alpha\beta}\gamma_\mu\gamma_5 s|K^*(P,\lambda)\rangle &= f^{\parallel}_K m_{K^*}\zeta^{\parallel}_{3K^*}\left\{e^{(\lambda)}_\alpha\left(P_\beta P_\mu - \frac{1}{3}\,m^2_{K^*}g_{\beta\mu}\right) - (\alpha\leftrightarrow\beta)\right\}\\ &\quad + \frac{1}{3}\,f^{\parallel}_K m^3_{K^*}\zeta^{\parallel}_{4K^*}\left(e^{(\lambda)}_\alpha g_{\beta\mu} - e^{(\lambda)}_\beta g_{\alpha\mu}\right), \qquad (3.12)\\ \langle 0|\bar q g G_{\alpha\beta} i\gamma_\mu s|K^*(P,\lambda)\rangle &= f^{\parallel}_K m_{K^*}\kappa^{\parallel}_{3K^*}\left\{e^{(\lambda)}_\alpha\left(P_\beta P_\mu - \frac{1}{3}\,m^2_{K^*}g_{\beta\mu}\right) - (\alpha\leftrightarrow\beta)\right\}\\ &\quad + \frac{1}{3}\,f^{\parallel}_K m^3_{K^*}\kappa^{\parallel}_{4K^*}\left(e^{(\lambda)}_\alpha g_{\beta\mu} - e^{(\lambda)}_\beta g_{\alpha\mu}\right). \qquad (3.13)\end{aligned}$$

Here we adopt the generic notation that ζ are G-conserving and κ G-breaking parameters. ζ_3, κ_3 are twist-3 and ζ_4, κ_4 twist-4 parameters.

Taking the local limit of eqs. (3.1) and (3.2), and comparing with the above definitions, one obtains

$$\begin{aligned}\phi^{\parallel}_0 &= -\frac{1}{3}\,\zeta^{\parallel}_{3K^*} + \frac{1}{3}\,\zeta^{\parallel}_{4K^*}\,, &\quad \theta^{\parallel}_0 &= -\frac{1}{3}\,\kappa^{\parallel}_{3K^*} + \frac{1}{3}\,\kappa^{\parallel}_{4K^*}\,,\\ \psi^{\parallel}_0 &= \frac{2}{3}\,\kappa^{\parallel}_{3K^*} + \frac{1}{3}\,\kappa^{\parallel}_{4K^*}\,, &\quad \widetilde\psi^{\parallel}_0 &= \frac{2}{3}\,\zeta^{\parallel}_{3K^*} + \frac{1}{3}\,\zeta^{\parallel}_{4K^*}\,. \qquad (3.14)\end{aligned}$$

The results for $\phi^{\parallel}_0$ and $\widetilde\psi^{\parallel}_0$ agree with those obtained in ref. [16], the others are new. Note that the "twist-4" DAs receive contributions from both twist-3 and -4 operators. This is due to the fact that the standard counting of twist in terms of "good" and "bad" components introduced in ref. [25] differs from the definition of twist as "dimension minus spin" of an operator. See also the discussion in section 2.2 of ref. [15].

What about the scale-dependence of these parameters? The relevant local twist-4 operator mixes with operators of lower twist for $m_s \neq 0$. Neglecting $\mathcal{O}(m_s^2)$ corrections, the mixing is given by

$$(\bar q\gamma_\alpha\gamma_5 g\widetilde G_{\mu\alpha}s)^{\mu^2} = (\bar q\gamma_\alpha\gamma_5 g\widetilde G_{\mu\alpha}s)^{\mu_0^2}\left(1 - \frac{8}{9}\,\frac{\alpha_s}{\pi}\,\ln\,\frac{\mu^2}{\mu_0^2}\right) + \frac{1}{9}\,\frac{\alpha_s}{\pi}\,\ln\,\frac{\mu^2}{\mu_0^2}\,m_s\,[\partial_\mu(\bar q i s)]^{\mu_0^2}\,. \qquad (3.15)$$

The matrix element of the derivative operator on the right-hand side vanishes for vector mesons, so that $\zeta^{\parallel}_{4K^*}$ renormalises multiplicatively even for $m_s \neq 0$. Resumming the logarithm, to LO accuracy, one has

$$\zeta^{\parallel}_{4K^*}(\mu^2) = L^{32/(9\beta_0)}\zeta^{\parallel}_{4K^*}(\mu_0^2)\,, \qquad (3.16)$$

with $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$.

The scale dependence of $\kappa^{\parallel}_{4K^*}$ can most easily be derived by observing that this parameter is related to $a^{\parallel}_1$ and quark masses by the QCD EOM [19]:

$$\kappa^{\parallel}_{4K^*} = -\frac{3}{20}\,a^{\parallel}_1 - \frac{f^{\perp}_{K^*}}{f^{\parallel}_{K^*}}\,\frac{m_s - m_q}{4m_{K^*}} + \frac{m_s^2 - m_q^2}{2m^2_{K^*}}\,. \qquad (3.17)$$

Taking into account the known scale dependence of $a_1^\parallel$, $f_{K^*}^\perp$ and $m_{s,q}$, one obtains

$$\kappa_{4K^*}^\parallel(\mu^2) = \kappa_{4K^*}^\parallel(\mu_0^2) - \frac{3}{20}\left(L^{32/(9\beta_0)} - 1\right) a_1^\parallel(\mu_0^2)$$
$$- \left(L^{16/(3\beta_0)} - 1\right) \frac{f_{K^*}^\perp(\mu_0^2)}{f_{K^*}^\parallel} \frac{[m_s - m_q](\mu_0^2)}{4m_{K^*}} + \left(L^{8/\beta_0} - 1\right) \frac{[m_s^2 - m_q^2](\mu_0^2)}{2m_{K^*}^2}. \quad (3.18)$$

To NLO in conformal spin, the discussion becomes more involved. As explained in ref. [9], for massless quarks the corresponding contributions can be expressed in terms of matrix elements of the three possible G-parity-even local quark-antiquark-gluon operators of twist 4. These three operators are not independent, however, but related by the QCD EOM. One is left with only one new non-perturbative parameter, $\widetilde{\omega}_{4K^*}^\parallel$,[3] which can be defined as

$$\langle 0|\bar{q}\left[iD_\mu, g\tilde{G}_{\nu\xi}\right]\gamma_\xi\gamma_5 s - \frac{4}{9}(i\partial_\mu)\bar{q}g\tilde{G}_{\nu\xi}\gamma_\xi\gamma_5 s|K^*(P,\lambda)\rangle + (\mu \leftrightarrow \nu) =$$
$$= 2f_{K^*}^\parallel m_{K^*}^3 \widetilde{\omega}_{4K^*}^\parallel \left(e_\mu^{(\lambda)} P_\nu + e_\nu^{(\lambda)} P_\mu\right). \quad (3.19)$$

The scale dependence of $\widetilde{\omega}_{4K^*}^\parallel$, for massless quarks, is given by

$$\widetilde{\omega}_{4K^*}^\parallel(\mu^2) = L^{10/\beta_0}\,\widetilde{\omega}_{4K^*}^\parallel(\mu_0^2).$$

For massive quarks, the twist-4 operator mixes with operators of lower twist. These lower-twist operators have the same Dirac structure as in (3.15), but additional derivatives acting on the fields. This means that, in terms of DAs, $\widetilde{\Phi}_{4;K^*}^\parallel$ mixes with $\psi_{3;K^*}^\parallel$, eq. (2.13), which in turn mixes with the three-particle twist-3 DA $\widetilde{\Phi}_{3;K^*}^\parallel$, eq. (2.15), which itself mixes with twist-2 DAs [17]. As the numerical impact of the admixture of m_s times lower-twist parameters is negligible for all cases investigated so far (twist-3 and -4 pseudoscalar parameters [14] and twist-3 vector parameters [17]), we refrain from working out these relations.

For massive quarks, one has to distinguish between G-parity-conserving and G-parity-breaking contributions. G-parity-conserving corrections do not involve new operators, and the difference to the massless case is mainly due to corrections proportional to the meson mass. This case is described in detail in refs. [15, 16]. Here we just quote the results obtained in ref. [16]:

$$\begin{aligned}
\phi_1^\parallel &= \frac{1}{12}a_2^\parallel - \frac{5}{12}\zeta_{3K^*}^\parallel + \frac{3}{16}\widetilde{\omega}_{3K^*}^\parallel + \frac{1}{8}\omega_{3K^*}^\parallel + \frac{7}{2}\widetilde{\omega}_{4K^*}^\parallel,\\
\phi_2^\parallel &= -\frac{1}{12}a_2^\parallel + \frac{3}{4}\zeta_{3K^*}^\parallel + \frac{3}{16}\widetilde{\omega}_{3K^*}^\parallel - \frac{1}{8}\omega_{3K^*}^\parallel + 7\widetilde{\omega}_{4K^*}^\parallel,\\
\psi_1^\parallel &= -\frac{1}{4}a_2^\parallel - \frac{7}{12}\zeta_{3K^*}^\parallel + \frac{3}{8}\omega_{3K^*}^\parallel - \frac{21}{4}\widetilde{\omega}_{4K^*}^\parallel,\\
\widetilde{\psi}_2^\parallel &= \frac{2}{3}\zeta_{3K^*}^\parallel - \frac{9}{16}\widetilde{\omega}_{3K^*}^\parallel + \frac{21}{4}\widetilde{\omega}_{4K^*}^\parallel,
\end{aligned} \quad (3.20)$$

[3] In the notation of ref. [16], $\widetilde{\omega}_{4K^*}^\parallel = \zeta_4\omega_4^A$.

where the terms in $a_2^{\parallel}$, $\zeta_{3K^*}^{\parallel}$, $\omega_{3K^*}^{\parallel}$ and $\widetilde{\omega}_{3K^*}^{\parallel}$ are mass corrections. $\omega_{3K^*}^{\parallel}$ and $\widetilde{\omega}_{3K^*}^{\parallel}$ are twist-3 parameters and have been discussed in ref. [17].

The G-parity-breaking contributions, on the other hand, involve a different set of local operators and in particular

$$\bar{q}\gamma_z D_\xi g G^{\xi z} s = -g^2 \sum_{\psi=u,d,s} (\bar{q}\gamma_z t^a s)(\bar{\psi}\gamma_z t^a \psi)\,,$$

which determines the normalization and the leading conformal spin contribution to the DA $\Xi_{4;K^*}^{\parallel}(\underline{\alpha})$ defined in eq. (3.10). Hence, a complete treatment of G-parity-breaking corrections to twist-4 DAs requires also the inclusion of $\Xi_{4;K^*}^{\parallel}$. It is beyond the scope of this paper to work out the corresponding relations between the matrix elements of local operators and expansion coefficients. Instead, we adopt a different approach and estimate G-parity-breaking corrections of conformal spin $j = 4$ using the renormalon model of ref. [13]. The general idea of this technique is to estimate matrix elements of "genuine" twist-4 operators by the quadratically divergent contributions that appear when the matrix elements are defined using a hard UV cut-off, see ref. [13] for details and further references. In this way, three-particle twist-4 DAs can be expressed in terms of the leading-twist DA $\phi_{2;K^*}^{\parallel}$:

$$\begin{aligned}
\Phi_{4;K^*}^{\parallel,\mathrm{R}}(\underline{\alpha}) &= -\frac{1}{6}\,\zeta_{4K^*}^{\parallel}\left[\frac{\phi_{2;K^*}^{\parallel}(\alpha_1)}{1-\alpha_1} - \frac{\phi_{2;K^*}^{\parallel}(\bar{\alpha}_2)}{1-\alpha_2}\right],\\
\widetilde{\Phi}_{4;K^*}^{\parallel,\mathrm{R}}(\underline{\alpha}) &= \frac{1}{6}\,\zeta_{4K^*}^{\parallel}\left[\frac{\phi_{2;K^*}^{\parallel}(\alpha_1)}{1-\alpha_1} + \frac{\phi_{2;K^*}^{\parallel}(\bar{\alpha}_2)}{1-\alpha_2}\right],\\
\Psi_{4;K^*}^{\parallel,\mathrm{R}}(\underline{\alpha}) &= \frac{1}{3}\,\zeta_{4K^*}^{\parallel}\left[\frac{\alpha_2\phi_{2;K^*}^{\parallel}(\alpha_1)}{(1-\alpha_1)^2} - \frac{\alpha_1\phi_{2;K^*}^{\parallel}(\bar{\alpha}_2)}{(1-\alpha_2)^2}\right],\\
\widetilde{\Psi}_{4;K^*}^{\parallel,\mathrm{R}}(\underline{\alpha}) &= \frac{1}{3}\,\zeta_{4K^*}^{\parallel}\left[\frac{\alpha_2\phi_{2;K^*}^{\parallel}(\alpha_1)}{(1-\alpha_1)^2} + \frac{\alpha_1\phi_{2;K^*}^{\parallel}(\bar{\alpha}_2)}{(1-\alpha_2)^2}\right],\\
\Xi_{4;K^*}^{\parallel,\mathrm{R}}(\underline{\alpha}) &= -\frac{2}{3}\,\zeta_{4K^*}^{\parallel}\left[\frac{\alpha_2\,\phi_{2;K^*}^{\parallel}(\alpha_1)}{1-\alpha_1} - \frac{\alpha_1\,\phi_{2;K^*}^{\parallel}(\bar{\alpha}_2)}{1-\alpha_2}\right],
\end{aligned}\qquad (3.21)$$

where, in contrast to ref. [13], we do not assume that $\phi_{2;K^*}^{\parallel}(u)$ is symmetric under the exchange $u \leftrightarrow \bar{u}$.

The expressions in (3.21) do not rely on conformal expansion and contain the contributions of all conformal partial waves. Projecting onto the contributions with the lowest spin $j = 3, 4$, we obtain

$$
\begin{aligned}
&\psi_0^{\parallel,\mathrm{R}} = 0\,, && \psi_1^{\parallel,\mathrm{R}} = \frac{7}{12}\zeta_{4K^*}^{\parallel}\,, && \psi_2^{\parallel,\mathrm{R}} = -\frac{7}{20}a_1^{\parallel}\zeta_{4K^*}^{\parallel},\\
&\widetilde{\psi}_0^{\parallel,\mathrm{R}} = \frac{1}{3}\zeta_{4K^*}^{\parallel}, && \widetilde{\psi}_1^{\parallel,\mathrm{R}} = \frac{7}{4}a_1^{\parallel}\zeta_{4K^*}^{\parallel}, && \widetilde{\psi}_2^{\parallel,\mathrm{R}} = -\frac{7}{12}\zeta_{4K^*}^{\parallel},\\
&\phi_0^{\parallel,\mathrm{R}} = \frac{1}{3}\zeta_{4K^*}^{\parallel}, && \phi_1^{\parallel,\mathrm{R}} = -\frac{7}{18}\zeta_{4K^*}^{\parallel}, && \phi_2^{\parallel,\mathrm{R}} = -\frac{7}{9}\zeta_{4K^*}^{\parallel},\\
&\theta_0^{\parallel,\mathrm{R}} = 0, && \theta_1^{\parallel,\mathrm{R}} = -\frac{7}{10}a_1^{\parallel}\zeta_{4K^*}^{\parallel}, && \theta_2^{\parallel,\mathrm{R}} = \frac{7}{5}a_1^{\parallel}\zeta_{4K^*}^{\parallel}\,,\\
&\xi_0^{\parallel,\mathrm{R}} = \frac{1}{5}a_1^{\parallel}\zeta_{4K^*}^{\parallel}\,. && &&
\end{aligned}
\tag{3.22}
$$

These results have to be compared with those in eqs. (3.14), (3.20) in the limit that all contributions from twist-2 and twist-3 parameters are set to 0. It follows that in the renormalon model

$$
\widetilde{\omega}_{4K^*}^{\parallel} = -\frac{1}{9}\,, \tag{3.23}
$$

which has the same sign as, but is larger than the result from a QCD sum rule calculation, see section 5. Also note that in the renormalon model $\psi_0^{\parallel} = \theta_0^{\parallel} = 0$, in contrast to (3.14). This is due to the fact that the contribution in $\kappa_{4K^*}^{\parallel}$ in (3.14) is obtained as the matrix element of the operator (3.13) which vanishes by the EOM (up to a total derivative), see (3.17). Therefore, against appearances, this contribution has to be interpreted as "kinematic" mass correction induced by the non-vanishing K^*-meson mass rather than a genuine twist-4 effect. The complete expressions for the G-odd parameters $\widetilde{\psi}_1^{\parallel}$, $\psi_2^{\parallel}$, $\theta_{1,2}^{\parallel}$ and $\xi_0^{\parallel}$ will contain mass corrections in terms of lower-twist parameters, whose determination is, as said before, beyond the scope of this paper.

We are now in a position to derive expressions for the chiral-even two-particle DAs of twist 4, $\phi_{4;K^*}^{\parallel}$ and $\psi_{4;K^*}^{\parallel}$, which are defined in eq. (2.8). From the operator relations collected in appendix A we obtain:

$$
\begin{aligned}
\psi_{4;K^*}^{\parallel}(u) = {}& \phi_{2;K^*}^{\parallel}(u) - 2\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2\, \frac{1}{\alpha_3}\left[2\Phi_{4;K^*}^{\parallel}(\underline{\alpha}) + \Psi_{4;K^*}^{\parallel}(\underline{\alpha})\right]\\
& - \frac{m_s - m_q}{m_{K^*}}\,\frac{f_{K^*}^{\perp}}{f_{K^*}^{\parallel}}\,\frac{d}{du}\,\psi_{3;K^*}^{\parallel}\,,
\end{aligned}
\tag{3.24}
$$

$$
\begin{aligned}
\frac{d}{du}\,\phi_{4;K^*}^{\parallel} = {}& 2\xi\left(\psi_{4;K^*}^{\parallel}(u) - \phi_{2;K^*}^{\parallel}(u)\right) - 4\int_0^u dv\left[5\phi_{2;K^*}^{\parallel}(v) - 8\phi_{3;K^*}^{\perp}(v) + 3\psi_{4;K^*}^{\parallel}(v)\right]\\
& - 4\frac{d}{du}\int_0^u d\alpha_1\int_0^{\bar u} d\alpha_2\,\frac{1}{\alpha_3^2}\,(\alpha_1 - \alpha_2 - \xi)\left[2\Phi_{4;K^*}^{\parallel}(\underline{\alpha}) + \Psi_{4;K^*}^{\parallel}(\underline{\alpha})\right]\\
& + \frac{m_s + m_q}{m_{K^*}}\,\frac{f_{K^*}^{\perp}}{f_{K^*}^{\parallel}}\,\frac{d}{du}\,2\psi_{3;K^*}^{\parallel}\,.
\end{aligned}
\tag{3.25}
$$

The latter equation has to be integrated with the boundary conditions $\phi_{4;K^*}^{\parallel}(0) = 0 = \phi_{4;K^*}^{\parallel}(1)$ which implies the relation (3.17) between $a_1^{\parallel}$ and $\kappa_{4K^*}^{\parallel}$. The boundary condition arises from the conversion of the matrix element of eq. (A.1) for the K^* into a relation for $\phi^{\parallel}_{4;K^*}$. This derivation of (3.17) is equivalent to that given in ref. [19].

$\psi^{\parallel}_{4;K^*}$ corresponds to the projection $s=-\frac{1}{2}$ for both quark and antiquark and hence, in the absence of quark-mass corrections in $m_s \pm m_q$, has an expansion in terms of $C_n^{1/2}(\xi)$. It can be split into contributions from genuine twist-4 parameters which are defined in terms of local twist-4 operators and kinematic Wandzura-Wilczek-type and mass corrections:

$$\psi^{\parallel}_{4;K^*}(u) = \psi^{\parallel,\mathrm{T4}}_{4;K^*}(u) + \psi^{\parallel,\mathrm{WW}}_{4;K^*}(u)\,. \tag{3.26}$$

$\psi^{\parallel,\mathrm{WW}}_{4;K^*}$ contains corrections explicitly proportional to $m_s \pm m_q$, of which we only keep the leading terms in $(m_s \pm m_q)^1$, but neglect higher powers.[4] We then find

$$\begin{aligned}
\psi^{\parallel,\mathrm{T4}}_{4;K^*}(u) &= -\frac{20}{3}\,\zeta^{\parallel}_{4K^*} C_2^{1/2}(\xi) + \left(10\theta_1^{\parallel} - 5\theta_2^{\parallel}\right) C_3^{1/2}(\xi)\,,\\
\psi^{\parallel,\mathrm{WW}}_{4;K^*}(u) &= 1 + \left(12\kappa^{\parallel}_{4K^*} + \frac{9}{5}\,a_1^{\parallel}\right) C_1^{1/2}(\xi) + \left(-1 - \frac{2}{7}\,a_2^{\parallel} + \frac{40}{3}\,\zeta^{\parallel}_{3K^*}\right) C_2^{1/2}(\xi)\\
&\quad + \left(-\frac{9}{5}\,a_1^{\parallel} - \frac{20}{3}\,\kappa^{\parallel}_{3K^*} - \frac{16}{3}\,\kappa^{\parallel}_{4K^*}\right) C_3^{1/2}(\xi)\\
&\quad + \left(-\frac{27}{28}\,a_2^{\parallel} + \frac{5}{4}\,\zeta^{\parallel}_{3K^*} - \frac{15}{8}\,\omega^{\parallel}_{3K^*} - \frac{15}{16}\,\widetilde{\omega}^{\parallel}_{3K^*}\right) C_4^{1/2}(\xi)\\
&\quad + 6\,\frac{m_s - m_q}{m_{K^*}}\,\frac{f^{\perp}_{K^*}}{f^{\parallel}_{K^*}} \left\{\xi + a_1^{\perp}\,\frac{1}{2}\,(3\xi^2-1) + a_2^{\perp}\,\frac{1}{2}\,\xi(5\xi^2-3) + \frac{5}{2}\,\kappa^{\perp}_{3K^*}(3\xi^2-1)\right.\\
&\qquad\qquad \left. + \frac{5}{6}\,\omega^{\perp}_{3K^*}\xi(5\xi^2-3) - \frac{1}{16}\,\lambda^{\perp}_{3K^*}(35\xi^4 - 30\xi^2 + 3)\right\}
\end{aligned} \tag{3.27}$$

$\phi^{\parallel}_{4;K^*}$, on the other hand, has no regular conformal expansion and contains logarithms even in the chiral limit. We solve the integral relation (3.25) by substituting all DAs on the right-hand side by their conformal expansion to NLO, implementing the boundary condition $\phi^{\parallel}_{4;K^*}(0) = 0 = \phi^{\parallel}_{4;K^*}(1)$ by eliminating $\kappa^{\parallel}_{4K^*}$ in favour of $a_1^{\parallel}$ according to (3.17), and dropping terms in $(m_s \pm m_q)^n$ with $n>1$. Like with $\psi^{\parallel}_{4;K^*}$, we distinguish between genuine twist-4 and mass corrections and write

$$\phi^{\parallel}_{4;K^*}(u) = \phi^{\parallel,\mathrm{T4}}_{4;K^*}(u) + \phi^{\parallel,\mathrm{WW}}_{4;K^*}(u)\,. \tag{3.28}$$

We then find:

$$\begin{aligned}
\phi^{\parallel,\mathrm{T4}}_{4;K^*}(u) &= 30u^2\bar{u}^2 \left\{\frac{20}{9}\,\zeta^{\parallel}_{4K^*} + \left(-\frac{8}{15}\,\theta_1^{\parallel} + \frac{2}{3}\,\theta_2^{\parallel}\right) C_1^{5/2}(\xi)\right\}\\
&\quad -84\widetilde{\omega}^{\parallel}_{4K^*}\left\{\frac{1}{8}\,u\bar{u}\,(21-13\xi^2) + u^3(10-15u+6u^2)\ln u + \bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln\bar{u}\right\}\\
&\quad + 80\psi_2^{\parallel}\left\{u^3(2-u)\ln u - \bar{u}^3(2-\bar{u})\ln\bar{u} - \frac{1}{8}\,(3\xi^2-11)\right\},
\end{aligned}$$

[4]The terms in $(m_s \pm m_q)^2$ actually diverge for $u \to 0, 1$, which is however largely irrelevant for phenomenological applications.

$$
\begin{aligned}
\phi_{4;K^*}^{\parallel,\mathrm{WW}}(u) = 30u^2\bar{u}^2 \Bigg\{ & \frac{4}{5}\left(1 + \frac{1}{21}\,a_2^\parallel + \frac{10}{9}\,\zeta_{3K^*}^\parallel\right) + \left(\frac{17}{50}\,a_1^\parallel + \frac{2}{5}\,\widetilde{\lambda}_{3K^*}^\parallel - \frac{1}{5}\,\lambda_{3K^*}^\parallel\right) C_1^{5/2}(\xi) \\
& + \frac{1}{10}\left(\frac{9}{7}\,a_2^\parallel + \frac{1}{9}\,\zeta_{3K^*}^\parallel + \frac{7}{6}\,\omega_{3K^*}^\parallel - \frac{3}{4}\,\widetilde{\omega}_{3K^*}^\parallel\right) C_2^{5/2}(\xi)\Bigg\} \\
& + 2\left\{-2a_2^\parallel - \frac{14}{3}\,\zeta_{3K^*}^\parallel + 3\omega_{3K^*}^\parallel\right\}\Bigg\{\frac{1}{8}\,u\bar{u}\,(21 - 13\xi^2) \\
& \qquad + u^3(10 - 15u + 6u^2)\ln u + \bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u}\Bigg\} \\
& + 4\left\{a_1^\parallel - \frac{40}{3}\,\kappa_{3K^*}^\parallel\right\}\left\{u^3(2-u)\ln u - \bar{u}^3(2-\bar{u})\ln\bar{u} - \frac{1}{8}\,(3\xi^2 - 11)\right\} \\
& + \frac{m_s + m_q}{m_{K^*}}\,\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\,6u\bar{u}\Bigg\{2(3 + 16a_2^\perp) + \frac{10}{3}(\kappa_{3K^*}^\perp - a_1^\perp)C_1^{3/2}(\xi) \\
& \qquad + \left(\frac{5}{9}\,\omega_{3K^*}^\perp - a_2^\perp\right) C_2^{3/2}(\xi) - \frac{1}{10}\,\lambda_{3K^*}^\perp C_3^{3/2}(\xi)\Bigg\} \\
& + 24\,\frac{m_s + m_q}{m_{K^*}}\,\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\,\left\{(1 - 3a_1^\perp + 6a_2^\perp)u^2\ln u + (1 + 3a_1^\perp + 6a_2^\perp)\bar{u}^2\ln\bar{u}\right\} \\
& + \frac{m_s - m_q}{m_{K^*}}\,\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\,6u\bar{u}\Bigg\{-\left(10\kappa_{3K^*}^\perp + \frac{82}{5}\,a_1^\perp\right) C_1^{3/2}(\xi) \\
& \qquad + 20\left(\frac{10}{189} + \frac{1}{3}\,a_2^\perp - \frac{1}{21}\,\omega_{3K^*}^\perp\right) C_2^{3/2}(\xi) + \left(\frac{7}{54}\,\lambda_{3K^*}^\perp + \frac{2}{5}\,a_1^\perp\right) C_3^{3/2}(\xi) \\
& \qquad + \left(\frac{1}{5}\,a_2^\perp - \frac{2}{315} - \frac{1}{21}\,\omega_{3K^*}^\perp\right) C_4^{3/2}(\xi) + \frac{2}{135}\,\lambda_{3K^*}^\perp C_5^{3/2}(\xi)\Bigg\} \\
& + \frac{m_s - m_q}{m_{K^*}}\,\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\,\Big\{(5u^2 - 23 - 54a_1^\perp - 108a_2^\perp)\ln\bar{u} \\
& \qquad - (5\bar{u}^2 - 23 + 54a_1^\perp - 108a_2^\perp)\ln u\Big\}\,. \qquad (3.29)
\end{aligned}
$$

Recall that $\bar{u} = 1 - u$ and $\xi = 2u - 1$. The DAs for $\bar{K}^* = (q\bar{s})$ mesons are obtained by replacing u by $1 - u$. Both (3.26) and (3.28) agree, for the ρ meson, with the results obtained in ref. [16].

The above expressions provide a self-consistent model of the twist-4 DAs which includes the first three terms of the conformal expansion. As mentioned above, one shortcoming of the model is that G-parity-breaking terms in $\psi_2^\parallel$ and $\theta_{1,2}^\parallel$ are not known exactly, but only in the renormalon model, eq. (3.22), which misses meson mass corrections. Numerically, the neglected parameters may be of the same size as the included ones.

An estimate of the size of higher orders in the conformal expansion can be obtained using the full renormalon model for $\psi_{4;K^*}^{\parallel,\mathrm{T4}}$ and $\phi_{4;K^*}^{\parallel,\mathrm{T4}}$. In this case, one has [13]

$$
\begin{aligned}
\psi_{4;K^*}^{\parallel,\mathrm{T4,R}}(u) &= -\frac{2}{3}\,\zeta_{4K^*}^\parallel\,\frac{d}{du}\left\{u\int_u^1 dv\,\frac{\phi_{2;K^*}^\parallel(v)}{v^2} - \bar{u}\int_0^u dv\,\frac{\phi_{2;K^*}^\parallel(v)}{\bar{v}^2}\right\}, \\
\phi_{4;K^*}^{\parallel,\mathrm{T4,R}}(u) &= \frac{8}{3}\,\zeta_{4K^*}^\parallel\left\{\int_0^u dv\,\frac{\phi_{2;K^*}^\parallel(v)}{\bar{v}^2}\left[\bar{u} + \bar{u}^2 + (u - v)\ln\frac{u - v}{\bar{v}}\right]\right. \\
&\qquad \left. + \int_u^1 dv\,\frac{\phi_{2;K^*}^\parallel(v)}{v^2}\left[u + u^2 + (v - u)\ln\frac{v - u}{v}\right]\right\}. \qquad (3.30)
\end{aligned}
$$

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As explained in ref. [13], the renormalon model does not take into account the damping of higher conformal-spin contributions by the increasing anomalous dimensions and, therefore, only provides an upper bound for their contribution. The most important effect of these corrections is to significantly enhance the end-point behaviour of higher-twist DAs in some cases, which can be important in phenomenological applications, for instance $D \to (\pi, K)$ form factors [26].

4. Chiral-odd distributions

The analysis of chiral-odd DAs proceeds along similar lines and, except for the inclusion of G-odd contributions, replicates that performed in ref. [16]. We first define the relevant three-particle DAs:

$$
\begin{aligned}
&\langle 0|\bar{q}(z)\sigma_{\alpha\beta} g G_{\mu\nu}(vz) s(-z)|K^*(P,\lambda)\rangle \\
&= f_{K^*}^\perp m_{K^*}^2 \frac{e^{(\lambda)} z}{2(pz)} [p_\alpha p_\mu g_{\beta\nu}^\perp - p_\beta p_\mu g_{\alpha\nu}^\perp - p_\alpha p_\nu g_{\beta\mu}^\perp + p_\beta p_\nu g_{\alpha\mu}^\perp] \Phi_{3;K^*}^\perp(v,pz) \\
&\quad + f_{K^*}^\perp m_{K^*}^2 [p_\alpha e_{\perp\mu}^{(\lambda)} g_{\beta\nu}^\perp - p_\beta e_{\perp\mu}^{(\lambda)} g_{\alpha\nu}^\perp - p_\alpha e_{\perp\nu}^{(\lambda)} g_{\beta\mu}^\perp + p_\beta e_{\perp\nu}^{(\lambda)} g_{\alpha\mu}^\perp] \Phi_{4;K^*}^{\perp(1)}(v,pz) \\
&\quad + f_{K^*}^\perp m_{K^*}^2 [p_\mu e_{\perp\alpha}^{(\lambda)} g_{\beta\nu}^\perp - p_\mu e_{\perp\beta}^{(\lambda)} g_{\alpha\nu}^\perp - p_\nu e_{\perp\alpha}^{(\lambda)} g_{\beta\mu}^\perp + p_\nu e_{\perp\beta}^{(\lambda)} g_{\alpha\mu}^\perp] \Phi_{4;K^*}^{\perp(2)}(v,pz) \\
&\quad + \frac{f_{K^*}^\perp m_{K^*}^2}{pz} [p_\alpha p_\mu e_{\perp\beta}^{(\lambda)} z_\nu - p_\beta p_\mu e_{\perp\alpha}^{(\lambda)} z_\nu - p_\alpha p_\nu e_{\perp\beta}^{(\lambda)} z_\mu + p_\beta p_\nu e_{\perp\alpha}^{(\lambda)} z_\mu] \Phi_{4;K^*}^{\perp(3)}(v,pz) \\
&\quad + \frac{f_{K^*}^\perp m_{K^*}^2}{pz} [p_\alpha p_\mu e_{\perp\nu}^{(\lambda)} z_\beta - p_\beta p_\mu e_{\perp\nu}^{(\lambda)} z_\alpha - p_\alpha p_\nu e_{\perp\mu}^{(\lambda)} z_\beta + p_\beta p_\nu e_{\perp\mu}^{(\lambda)} z_\alpha] \Phi_{4;K^*}^{\perp(4)}(v,pz) + \ldots
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
\langle 0|\bar{q}(z) g G_{\mu\nu}(vz) s(-z)|K^*(P,\lambda)\rangle &= i f_{K^*}^\perp m_{K^*}^2 [e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu] \Psi_{4;K^*}^\perp(v,pz) + \ldots, \\
\langle 0|\bar{q}(z) i g \widetilde{G}_{\mu\nu}(vz)\gamma_5 s(-z)|K^*(P,\lambda)\rangle &= i f_{K^*}^\perp m_{K^*}^2 [e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu] \widetilde{\Psi}_{4;K^*}^\perp(v,pz) + \ldots, \\
\langle 0|\bar{q}(z)\sigma_{\mu\nu} g D_\alpha G_{\alpha\beta}(vz) s(-z)|K^*(P,\lambda)\rangle &= i f_{K^*}^\perp m_{K^*}^2 \left[e_{\perp\mu}^{(\lambda)} p_\nu - e_{\perp\nu}^{(\lambda)} p_\mu\right] p_\beta\, \Xi_{4;K^*}^\perp(v,pz) + \ldots
\end{aligned} \tag{4.2}
$$

The twist-3 DA $\Phi_{3;K^*}^\perp$ was already mentioned in section 2; the twist-4 DAs are related to those defined in ref. [16] by $\Phi_{4;K^*}^{\perp(i)} = T_i^{(4)}$, $\Psi_{4;K^*}^\perp = S$ and $\widetilde{\Psi}_{4;K^*}^\perp = \widetilde{S}$; $\Xi_{4;K^*}^\perp$ was first introduced in ref. [13]. As usual, the dots denote terms of higher twist.

As the matrix element in (4.1) is G-odd, the DAs $\Phi_{4;K^*}^{\perp(i)}$ are, in the limit of equal mass quarks, antisymmetric under the exchange of α_1 and α_2, whereas $\Psi_{4;K^*}^\perp$ and $\widetilde{\Psi}_{4;K^*}^\perp$ are symmetric. In order to resolve the conformal structure of $\Phi_{4;K^*}^{\perp(i)}$, it is useful to exploit the fact that $\sigma_{\mu\nu}\gamma_5$ is not independent of $\sigma_{\mu\nu}$, and to define the "dual" matrix element

$$
\langle 0|\bar{q}(z) i\sigma_{\alpha\beta}\gamma_5 g\widetilde{G}_{\mu\nu}(vz) s(-z)|K^*\rangle = \text{r.h.s. of (4.1) with } \Phi_{4;K^*}^{\perp(i)} \to \widetilde{\Phi}_{4;K^*}^{\perp(i)}. \tag{4.3}
$$

One easily finds

$$
\begin{aligned}
\Phi_{3;K^*}^\perp &= \widetilde{\Phi}_{3;K^*}^\perp, \quad \widetilde{\Phi}_{4;K^*}^{\perp(1)} = -\Phi_{4;K^*}^{\perp(3)}, \quad \widetilde{\Phi}_{4;K^*}^{\perp(2)} = -\Phi_{4;K^*}^{\perp(4)}, \\
\widetilde{\Phi}_{4;K^*}^{\perp(3)} &= -\Phi_{4;K^*}^{\perp(1)}, \quad \widetilde{\Phi}_{4;K^*}^{\perp(4)} = -\Phi_{4;K^*}^{\perp(2)}.
\end{aligned} \tag{4.4}
$$

$\Phi^{\perp(1)}_{4;K^*}$ and $\widetilde{\Phi}^{\perp(1)}_{4;K^*}$ correspond to the Lorentz spin projection $s = +1/2$ for both quark fields and $s = 0$ for the gluon. Hence the conformal expansion reads, to NLO:

$$\begin{aligned} \Phi^{\perp(1)}_{4;K^*}(\underline{\alpha}) &= 120\alpha_1\alpha_2\alpha_3[\phi_0^\perp + \phi_1^\perp(\alpha_1-\alpha_2) + \phi_2^\perp(3\alpha_3-1)]\,, \\ -\Phi^{\perp(3)}_{4;K^*}(\underline{\alpha}) = \widetilde{\Phi}^{\perp(1)}_{4;K^*}(\underline{\alpha}) &= 120\alpha_1\alpha_2\alpha_3[\widetilde{\phi}_0^\perp + \widetilde{\phi}_1^\perp(\alpha_1-\alpha_2) + \widetilde{\phi}_2^\perp(3\alpha_3-1)]\,. \end{aligned} \tag{4.5}$$

Here $\phi^\perp_{0,2}$ are G-violating and $\phi_1^\perp$ is G-conserving; the same applies to $\widetilde{\phi}_i^\perp$.

The conformal expansion of $\Xi^\perp_{4;K^*}$ starts at $j = 4$ and reads, to NLO accuracy:

$$\Xi^\perp_{4;K^*}(\underline{\alpha}) = 840\alpha_1\alpha_2\alpha_3^2\xi_0^\perp\,. \tag{4.6}$$

The remaining DAs do not correspond to fixed values of the Lorentz-spin projection s. Like in the chiral-even case, we introduce auxiliary amplitudes with a regular conformal expansion:

$$\begin{aligned} \langle 0|\bar q(z)\gamma_z\gamma_p gG_{\mu\nu}(vz)s(-z)|K^*(P,\lambda)\rangle &= if^\perp_{K^*}m^2_{K^*}(pz)[e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]\Psi^{\uparrow\downarrow}(v,pz), \\ \langle 0|\bar q(z)\gamma_z\gamma_p i\gamma_5 g\widetilde{G}_{\mu\nu}(vz)s(-z)|K^*(P,\lambda)\rangle &= if^\perp_{K^*}m^2_{K^*}(pz)[e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]\widetilde{\Psi}^{\uparrow\downarrow}(v,pz), \end{aligned} \tag{4.7}$$

and, similarly, two more distributions $\Psi^{\downarrow\uparrow}$ and $\widetilde{\Psi}^{\downarrow\uparrow}$ by replacing $\gamma_z\gamma_p \to \gamma_p\gamma_z$. The DAs in (4.1) and (4.2) are then given by:

$$\begin{aligned} \Psi^\perp_{4;K^*}(\underline{\alpha}) &= \frac{1}{2}\left(\Psi^{\uparrow\downarrow}(\underline{\alpha}) + \Psi^{\downarrow\uparrow}(\underline{\alpha})\right), & \widetilde{\Psi}^\perp_{4;K^*}(\underline{\alpha}) &= \frac{1}{2}\left(\widetilde{\Psi}^{\uparrow\downarrow}(\underline{\alpha}) + \widetilde{\Psi}^{\downarrow\uparrow}(\underline{\alpha})\right), \\ \Phi^{\perp(4)}_{4;K^*}(\underline{\alpha}) &= \frac{1}{2}\left(\Psi^{\uparrow\downarrow}(\underline{\alpha}) - \Psi^{\downarrow\uparrow}(\underline{\alpha})\right), & -\Phi^{\perp(2)}_{4;K^*}(\underline{\alpha}) &= \widetilde{\Phi}^{\perp(4)}_{4;K^*}(\underline{\alpha}) = \frac{1}{2}\left(\widetilde{\Psi}^{\uparrow\downarrow}(\underline{\alpha}) - \widetilde{\Psi}^{\downarrow\uparrow}(\underline{\alpha})\right). \end{aligned} \tag{4.8}$$

$\Psi^{\uparrow\downarrow}$ and $\Psi^{\downarrow\uparrow}$ have a regular expansion in terms of conformal polynomials, to wit:

$$\begin{aligned} \Psi^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_2\alpha_3^2\left[\psi_0^{\uparrow\downarrow} + \psi_1^{\uparrow\downarrow}(\alpha_3-3\alpha_1) + \psi_2^{\uparrow\downarrow}\left(\alpha_3 - \frac{3}{2}\alpha_2\right)\right], \\ \Psi^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_1\alpha_3^2\left[\psi_0^{\downarrow\uparrow} + \psi_1^{\downarrow\uparrow}(\alpha_3-3\alpha_2) + \psi_2^{\downarrow\uparrow}\left(\alpha_3 - \frac{3}{2}\alpha_1\right)\right]. \end{aligned} \tag{4.9}$$

Again G-parity ensures that for the ρ and ϕ mesons $\psi_i^{\uparrow\downarrow} \equiv \psi_i^{\downarrow\uparrow}$. For K^*, we write

$$\psi_i^{\uparrow\downarrow} = \psi_i^\perp + \theta_i^\perp, \qquad \psi_i^{\downarrow\uparrow} = \psi_i^\perp - \theta_i^\perp, \tag{4.10}$$

where the $\theta_i^\perp$ correspond to SU(3)-breaking corrections that also violate G-parity. Introducing an analogous decomposition of $\widetilde{\Psi}^{\uparrow\downarrow}$ and $\widetilde{\Psi}^{\downarrow\uparrow}$ in terms of $\widetilde{\psi}_i^\perp$ and $\widetilde{\theta}_i^\perp$, we then find

$$\begin{aligned} \Psi^\perp_{4;K^*}(\underline{\alpha}) &= 30\alpha_3^2\left\{\psi_0^\perp(1-\alpha_3) + \psi_1^\perp\left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right]\right. \\ &\quad \left. + \psi_2^\perp\left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\theta_0^\perp + \alpha_3\theta_1^\perp + \frac{1}{2}(5\alpha_3-3)\theta_2^\perp\right]\right\}, \\ \widetilde{\Psi}^\perp_{4;K^*}(\underline{\alpha}) &= 30\alpha_3^2\left\{\widetilde{\psi}_0^\perp(1-\alpha_3) + \widetilde{\psi}_1^\perp\left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right]\right. \\ &\quad \left. + \widetilde{\psi}_2^\perp\left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\widetilde{\theta}_0^\perp + \alpha_3\widetilde{\theta}_1^\perp + \frac{1}{2}(5\alpha_3-3)\widetilde{\theta}_2^\perp\right]\right\}, \end{aligned}$$

$$
\begin{aligned}
\Phi^{\perp(2)}_{4;K^*}(\underline{\alpha}) &= -30\alpha_3^2 \Big\{ \widetilde{\theta}^\perp_0 (1-\alpha_3) + \widetilde{\theta}^\perp_1 \left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right] \\
&\quad + \widetilde{\theta}^\perp_2 \left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\widetilde{\psi}^\perp_0 + \alpha_3\widetilde{\psi}^\perp_1 + \frac{1}{2}(5\alpha_3-3)\widetilde{\psi}^\perp_2\right]\Big\}, \\
\Phi^{\perp(4)}_{4;K^*}(\underline{\alpha}) &= 30\alpha_3^2 \Big\{ \theta^\perp_0 (1-\alpha_3) + \theta^\perp_1 \left[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\right] \\
&\quad + \theta^\perp_2 \left[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\right] - (\alpha_1-\alpha_2)\left[\psi^\perp_0 + \alpha_3\psi^\perp_1 + \frac{1}{2}(5\alpha_3-3)\psi^\perp_2\right]\Big\}.
\end{aligned} \tag{4.11}
$$

Our next task is to relate $\psi_i^\perp$ and $\theta_i^\perp$ to local matrix elements. To LO in the conformal expansion, the relevant matrix elements are [16]

$$
\begin{aligned}
\langle 0|\bar q g G_{\alpha\beta} s|K^*(P,\lambda)\rangle &= i f^\perp_K m^2_{K^*} \zeta^\perp_{4K^*} \left(\epsilon^{(\lambda)}_\alpha P_\beta - \epsilon^{(\lambda)}_\beta P_\alpha\right), \\
\langle 0|\bar q g \widetilde{G}_{\alpha\beta} i\gamma_5 s|K^*(P,\lambda)\rangle &= i f^\perp_K m^2_{K^*} \widetilde{\zeta}^\perp_{4K^*} \left(\epsilon^{(\lambda)}_\alpha P_\beta - \epsilon^{(\lambda)}_\beta P_\alpha\right), \\
\langle 0|\bar q g G_{\alpha\mu}\sigma_{\beta\mu} s|K^*(P,\lambda)\rangle &= f^\perp_K m^2_{K^*} \left\{\frac{1}{2}\kappa^\perp_{3K^*}\left(\epsilon^{(\lambda)}_\alpha P_\beta + \epsilon^{(\lambda)}_\beta P_\alpha\right) + \kappa^\perp_{4K^*}\left(\epsilon^{(\lambda)}_\alpha P_\beta - \epsilon^{(\lambda)}_\beta P_\alpha\right)\right\},
\end{aligned} \tag{4.12}
$$

where again ζ denotes G-parity-conserving parameters and κ G-parity-breaking ones. The twist-3 parameter $\kappa^\perp_{3K^*}$ was investigated in ref. [17]. Taking the local limit of (4.1) and (4.2) and comparing with the above, one finds

$$
\begin{aligned}
\psi^\perp_0 &= \zeta^\perp_{4K^*}, & \widetilde{\psi}^\perp_0 &= \widetilde{\zeta}^\perp_{4K^*}, \\
\phi^\perp_0 &= \frac{1}{6}\kappa^\perp_{3K^*} + \frac{1}{3}\kappa^\perp_{4K^*}, & \widetilde{\phi}^\perp_0 &= \frac{1}{6}\kappa^\perp_{3K^*} - \frac{1}{3}\kappa^\perp_{4K^*}, \\
\theta^\perp_0 &= -\frac{1}{6}\kappa^\perp_{3K^*} - \frac{1}{3}\kappa^\perp_{4K^*}, & \widetilde{\theta}^\perp_0 &= -\frac{1}{6}\kappa^\perp_{3K^*} + \frac{1}{3}\kappa^\perp_{4K^*}.
\end{aligned} \tag{4.13}
$$

Whereas $\zeta^\perp_{4K^*}$, $\widetilde{\zeta}^\perp_{4K^*}$ and $\kappa^\perp_{3K^*}$ are independent parameters, $\kappa^\perp_{4K^*}$ depends on quark masses and $a^\perp_1$ by a relation that is analogous to eq. (3.17) and was obtained in ref. [21]:

$$
\kappa^\perp_{4K^*} = \frac{1}{10}\, a^\perp_1 + \frac{f^\parallel_{K^*}}{f^\perp_{K^*}}\,\frac{m_s - m_q}{12 m_{K^*}} - \frac{m_s^2 - m_q^2}{4 m^2_{K^*}}. \tag{4.14}
$$

Like with $\kappa^\parallel_{4K^*}$, the scale-dependence of $\kappa^\perp_{4K^*}$ follows from that of the parameters on the right-hand side and is given by

$$
\begin{aligned}
\kappa^\perp_{4K^*}(\mu^2) &= \kappa^\perp_{4K^*}(\mu_0^2) + \left(L^{8/(3\beta_0)} - 1\right)\frac{1}{10}\, a^\perp_1(\mu_0^2) + \left(L^{8/(3\beta_0)} - 1\right)\frac{f^\parallel_{K^*}}{f^\perp_{K^*}(\mu_0^2)}\,\frac{[m_s - m_q](\mu_0^2)}{12 m_{K^*}} \\
&\quad - \left(L^{8/\beta_0} - 1\right)\frac{[m_s^2 - m_q^2](\mu_0^2)}{4 m^2_{K^*}}
\end{aligned} \tag{4.15}
$$

with $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$. In the limit of zero quark mass, the parameters ζ^T_4, $\widetilde{\zeta}^T_4$ renormalise multiplicatively [23]:

$$
\begin{aligned}
\left(\zeta^T_4 + \widetilde{\zeta}^T_4\right)(\mu^2) &= L^{\gamma^+/\beta_0}\left(\zeta^T_4 + \widetilde{\zeta}^T_4\right)(\mu_0^2), & \gamma_+ &= 3C_A - \frac{8}{3}C_F, \\
\left(\zeta^T_4 - \widetilde{\zeta}^T_4\right)(\mu^2) &= L^{\gamma^-/\beta_0}\left(\zeta^T_4 - \widetilde{\zeta}^T_4\right)(\mu_0^2), & \gamma_- &= 4C_A - 4C_F.
\end{aligned} \tag{4.16}
$$

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Again, these simple scaling relations will receive corrections from terms proportional to the quark masses. These corrections are unknown, but based on the experience with pseudoscalar twist-4 matrix elements [14] we do not expect them to be large.

The calculation of the NLO G-even corrections is pretty involved and presented in detail in appendix B of ref. [16]. The upshot is that the six coefficients $\psi^\perp_{1,2}$, $\widetilde{\psi}^\perp_{1,2}$, $\phi^\perp_1$ and $\widetilde{\phi}^\perp_1$ involve three additional genuine twist-4 parameters $\langle\!\langle Q^{(1,3,5)}\rangle\!\rangle$:

$$\begin{aligned}
\phi^\perp_1 &= \frac{9}{44}\,a^\perp_2 + \frac{1}{8}\,\omega^\perp_{3K^*} + \frac{63}{220}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{119}{44}\,\langle\!\langle Q^{(3)}\rangle\!\rangle\,,\\
\widetilde{\phi}^\perp_1 &= -\frac{9}{44}\,a^\perp_2 + \frac{1}{8}\,\omega^\perp_{3K^*} - \frac{63}{220}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{35}{44}\,\langle\!\langle Q^{(3)}\rangle\!\rangle\,,\\
\psi^\perp_1 &= \frac{3}{44}\,a^\perp_2 + \frac{1}{12}\,\omega^\perp_{3K^*} + \frac{49}{110}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{7}{22}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{7}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle\,,\\
\widetilde{\psi}^\perp_1 &= -\frac{3}{44}\,a^\perp_2 + \frac{1}{12}\,\omega^\perp_{3K^*} - \frac{49}{110}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{7}{22}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{7}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle\,,\\
\psi^\perp_2 &= -\frac{3}{22}\,a^\perp_2 - \frac{1}{12}\,\omega^\perp_{3K^*} + \frac{28}{55}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{7}{11}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{14}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle\,,\\
\widetilde{\psi}^\perp_2 &= \frac{3}{22}\,a^\perp_2 - \frac{1}{12}\,\omega^\perp_{3K^*} - \frac{28}{55}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{7}{11}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{14}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle\,. \qquad (4.17)
\end{aligned}$$

The precise definition of $\langle\!\langle Q^{(i)}\rangle\!\rangle$ is given in ref. [16]. $\omega^\perp_{3K^*}$ is a twist-3 parameter and defined in ref. [17]. Existing numerical determinations of these parameters from QCD sum rules are far from being precise, so we decide to estimate them using the renormalon model of ref. [13] instead. The model entails the following expressions of the three-particle twist-4 DAs in terms of $\phi^\perp_{2;K^*}$:

$$\begin{aligned}
\Phi^{\perp(1),\mathrm{R}}_{4;K^*}(\underline{\alpha}) = -\Phi^{\perp(3),\mathrm{R}}_{4;K^*}(\underline{\alpha}) &= \zeta^\perp_{4K^*}\left[\frac{\alpha_2\phi^\perp_{2;K^*}(\alpha_1)}{(1-\alpha_1)^2} - \frac{\alpha_1\phi^\perp_{2;K^*}(\bar{\alpha}_2)}{(1-\alpha_2)^2}\right],\\
\Phi^{\perp(2),\mathrm{R}}_{4;K^*}(\underline{\alpha}) = \Phi^{\perp(4),\mathrm{R}}_{4;K^*}(\underline{\alpha}) &= -\frac{1}{2}\zeta^\perp_{4K^*}\left[\frac{\phi^\perp_{2;K^*}(\alpha_1)}{(1-\alpha_1)} - \frac{\phi^\perp_{2;K^*}(\bar{\alpha}_2)}{(1-\alpha_2)}\right],\\
\Psi^{\perp,\mathrm{R}}_{4;K^*}(\underline{\alpha}) = -\widetilde{\Psi}^{\perp,\mathrm{R}}_{4;K^*}(\underline{\alpha}) &= \frac{1}{2}\zeta^\perp_{4K^*}\left[\frac{\phi^\perp_{2;K^*}(\alpha_1)}{1-\alpha_1} + \frac{\phi^\perp_{2;K^*}(\bar{\alpha}_2)}{1-\alpha_2}\right],\\
\Xi^{\perp,\mathrm{R}}_{4;K^*}(\underline{\alpha}) &= -2\zeta^\perp_{4K^*}\left[\frac{\alpha_2}{1-\alpha_1}\,\phi^\perp_{2;K^*}(\alpha_1) - \frac{\alpha_1}{1-\alpha_2}\phi^\perp_{2;K^*}(\bar{\alpha}_2)\right], \qquad (4.18)
\end{aligned}$$

which is similar to the results for the chiral-even DAs, eq. (3.21). The model implies the following estimates for the G-conserving twist-4 parameters in (4.13) and (4.17):

$$\widetilde{\zeta}^{\perp,\mathrm{R}}_{4K^*} = -\zeta^\perp_{4K^*}\,,\quad \langle\!\langle Q^{(1)}\rangle\!\rangle^{\mathrm{R}} = -\frac{10}{3}\zeta^\perp_{4K^*}\,,\quad \langle\!\langle Q^{(3)}\rangle\!\rangle^{\mathrm{R}} = -\zeta^\perp_{4K^*}\,,\quad \langle\!\langle Q^{(5)}\rangle\!\rangle^{\mathrm{R}} = 0\,. \qquad (4.19)$$

The G-parity-violating parameters are given by:

$$\begin{aligned}
&\phi^{\perp,\mathrm{R}}_0 = 0\,, && \widetilde{\phi}^{\perp,\mathrm{R}}_0 = 0\,, && \phi^{\perp,\mathrm{R}}_2 = -\frac{21}{20}\,\zeta^\perp_{4K^*}a^\perp_1\,, \quad \widetilde{\phi}^{\perp,\mathrm{R}}_2 = -\frac{21}{20}\,\zeta^\perp_{4K^*}a^\perp_1\,,\\
&\theta^{\perp,\mathrm{R}}_0 = 0\,, && \widetilde{\theta}^{\perp,\mathrm{R}}_0 = 0\,, && \theta^{\perp,\mathrm{R}}_1 = -\frac{21}{10}\,\zeta^\perp_{4K^*}a^\perp_1\,, \quad \widetilde{\theta}^{\perp,\mathrm{R}}_1 = \frac{21}{10}\,\zeta^\perp_{4K^*}a^\perp_1\,,\\
&\theta^{\perp,\mathrm{R}}_2 = \frac{21}{5}\,\zeta^\perp_{4K^*}a^\perp_1\,, && \widetilde{\theta}^{\perp,\mathrm{R}}_2 = -\frac{21}{5}\,\zeta^\perp_{4K^*}a^\perp_1\,, && \xi^{\perp,\mathrm{R}}_0 = \frac{3}{5}\zeta^\perp_{4K^*}a^\perp_1\,. \qquad (4.20)
\end{aligned}$$

As with chiral-even distributions, the above results provide an estimate only for the genuine twist-4 contributions, but miss any mass corrections in terms of lower-twist parameters. This is also the reason why $\phi_0^\perp$, $\theta_0^\perp$, $\widetilde{\phi}_0^\perp$ and $\widetilde{\theta}_0^\perp$ vanish in the renormalon model, in contrast to eq. (4.13).

Let us now turn to the two-particle twist-4 DAs $\phi_{4;K^*}^\perp$ and $\psi_{4;K^*}^\perp$ defined in eq. (2.9). From the operator relations given in appendix A we obtain:

$$\begin{aligned}
\psi_{4;K^*}^\perp(u) &= -\phi_{2;K^*}^\perp(u) + 2\psi_{3;K^*}^\parallel(u) - 2\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2 \left\{ \frac{1}{\alpha_3^2}(\alpha_1-\alpha_2-\xi)\Psi_{4;K^*}^\perp(\underline{\alpha}) \right. \\
&\qquad \left. -\frac{1}{\alpha_3}\left(\Phi_{4;K^*}^{\perp(2)}(\underline{\alpha}) - \Phi_{4;K^*}^{\perp(3)}(\underline{\alpha})\right)\right\} + 2\frac{f_{K^*}^\parallel}{f_{K^*}^\perp}\frac{m_s+m_q}{m_{K^*}}\phi_{3;K^*}^\perp(u)\,, \qquad (4.21) \\
\frac{d}{du}\phi_{4;K^*}^\perp(u) &= -2\xi\left\{\psi_{4;K^*}^\perp(u) + \phi_{2;K^*}^\perp(u)\right\} + 8\int_0^u dv \{\psi_{4;K^*}^\perp(v) - \phi_{2;K^*}^\perp(v)\} \\
&\quad - \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2 \frac{4}{\alpha_3}\left\{ \frac{\alpha_1-\alpha_2-\xi}{\alpha_3}\left[\Phi_{4;K^*}^{\perp(2)}(\underline{\alpha}) - \Phi_{4;K^*}^{\perp(3)}(\underline{\alpha})\right] - \Psi_{4;K^*}^\perp(\underline{\alpha})\right\} \\
&\quad + 4\frac{f_{K^*}^\parallel}{f_{K^*}^\perp}\frac{m_s-m_q}{m_{K^*}}\phi_{3;K^*}^\perp(u)\,. \qquad (4.22)
\end{aligned}$$

The boundary condition for $\phi_{4;K^*}^\perp$ is $\phi_{4;K^*}^\perp(0) = 0 = \phi_{4;K^*}^\perp(1)$, which implies the relation (4.14). In the renormalon model, one obtains exact expressions for these DAs [13]:

$$\begin{aligned}
\psi_{4;K^*}^{\perp,\mathrm{T4,R}}(u) &= 0\,, \\
\phi_{4;K^*}^{\perp.\mathrm{T4,R}}(u) &= 8\zeta_{4K^*}^\perp \left[\int_0^u dv \left(\bar u + (u-v)\ln\frac{u-v}{\bar v}\right)\frac{\phi_{2;K^*}^\perp(v)}{\bar v^2} \right. \\
&\qquad \left. + \int_u^1 dv\left(u + (v-u)\ln\frac{v-u}{v}\right)\frac{\phi_{2;K^*}^\perp(v)}{v^2}\right]. \qquad (4.23)
\end{aligned}$$

Like the chiral-even DA $\psi_{4;K^*}^\parallel$, $\psi_{4;K^*}^\perp$ corresponds to the projection $s = -\frac{1}{2}$ for both quark and antiquark and hence, in the absence of quark-mass corrections in $m_s \pm m_q$, has an expansion in terms of $C_n^{1/2}(\xi)$. The full $\psi_{4;K^*}^\perp$ contains corrections explicitly proportional to $m_s \pm m_q$, of which we only keep the leading term in $(m_s \pm m_q)^1$. To NLO in the conformal expansion, (4.21) yields:

$$\begin{aligned}
\psi_{4;K^*}^\perp(u) &= 1 + \left(12\kappa_{4K^*}^\perp - \frac{3}{5}a_1^\perp\right)C_1^{1/2}(\xi) + \left(-1 + \frac{3}{7}a_2^\perp - 10\left\{\zeta_{4K^*}^\perp + \widetilde{\zeta}_{4K^*}^\perp\right\}\right)C_2^{1/2}(\xi) \\
&\quad + \left\{-5\kappa_{3K^*}^\perp - 12\kappa_{4K^*}^\perp - \frac{1}{3}\lambda_{3K^*}^\perp + \frac{3}{5}a_1^\perp + 5\left[\theta_1^\perp + \widetilde{\theta}_1^\perp - \frac{1}{2}\left(\theta_2^\perp + \widetilde{\theta}_2^\perp\right)\right]\right\}C_3^{1/2}(\xi) \\
&\quad + \left(-\frac{5}{4}\omega_{3K^*}^\perp - \frac{3}{7}a_2^\perp\right)C_4^{1/2}(\xi) + \frac{1}{3}\lambda_{3K^*}^\perp C_5^{1/2}(\xi) \\
&\quad + \frac{m_s+m_s}{m_{K^*}}\frac{f_{K^*}^\parallel}{f_{K^*}^\perp}\left\{3\left(1+6a_2^\parallel\right) + 3a_1^\parallel C_1^{1/2}(\xi) + 5\left(4\zeta_{3K^*}^\parallel - 3a_2^\parallel\right)C_2^{1/2}(\xi)\right. \\
&\qquad \left. + 5\left(4\kappa_{3K^*}^\parallel - \frac{3}{4}\lambda_{3K^*}^\parallel + \frac{3}{2}\widetilde{\lambda}_{3K^*}^\parallel\right)C_3^{1/2}(\xi) + \frac{15}{4}\left(2\omega_{3K^*}^\parallel - \widetilde{\omega}_{3K^*}^\parallel\right)C_4^{1/2}(\xi)\right\}
\end{aligned}$$

$$+ 6\,\frac{m_s+m_s}{m_{K^*}}\,\frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}}\,\left\{\left(1-3a_1^{\parallel}+6a_2^{\parallel}\right)u\ln u+\left(1+3a_1^{\parallel}+6a_2^{\parallel}\right)\bar u\ln\bar u\right\}$$
$$- 6\,\frac{m_s-m_s}{m_{K^*}}\,\frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}}\,u\bar u\left(9a_1^{\parallel}+10\xi a_2^{\parallel}\right)$$
$$+ 6\,\frac{m_s-m_s}{m_{K^*}}\,\frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}}\,\left\{\left(1-3a_1^{\parallel}+6a_2^{\parallel}\right)u\ln u-\left(1+3a_1^{\parallel}+6a_2^{\parallel}\right)\bar u\ln\bar u\right\}. \tag{4.24}$$

Recall that $\bar u = 1-u$ and $\xi = 2u-1$. The above expression refers to a $K^* = (s\bar q)$ meson; for $\bar K^* = (q\bar s)$, one has to replace u by $1-u$.

The explicit formula for $\phi_{4;K^*}^{\perp}$ from (4.22) is very long and complicated, so we only give the result for $m_s \pm m_q \to 0$:

$$\begin{aligned}
\phi_{4;K^*}^{\perp}(u) = 30u^2\bar u^2 \Bigg\{ &\left(\frac{4}{3}\,\zeta_{4K^*}^{\perp}-\frac{8}{3}\,\widetilde\zeta_{4K^*}^{\perp}+\frac{2}{5}+\frac{4}{35}\,a_2^{\perp}\right)\\
&+\left(\frac{3}{25}\,a_1^{\perp}+\frac{1}{3}\,\kappa_{3K^*}^{\perp}-\frac{1}{45}\,\lambda_{3K^*}^{\perp}-\frac{1}{15}\,\theta_1^{\perp}+\frac{7}{30}\,\theta_2^{\perp}+\frac{1}{5}\,\widetilde\theta_1^{\perp}-\frac{3}{10}\,\widetilde\theta_2^{\perp}\right)C_1^{5/2}(\xi)\\
&+\left(\frac{3}{35}\,a_2^{\perp}+\frac{1}{60}\,\omega_{3K^*}^{\perp}\right)C_2^{5/2}(\xi)-\frac{4}{1575}\,\lambda_{3K^*}^{\perp}C_3^{5/2}(\xi)\Bigg\}\\
&+\left(5\kappa_{3K^*}^{\perp}-a_1^{\perp}-20\widetilde\phi_2^{\perp}\right)\left\{-4u^3(2-u)\ln u+4\bar u^3(2-\bar u)\ln\bar u+\frac{1}{2}\,u\bar u\xi(3\xi^2-11)\right\}\\
&+\left(2\omega_{3K^*}^{\perp}-\frac{36}{11}\,a_2^{\perp}-\frac{252}{55}\,\langle\!\langle Q^{(1)}\rangle\!\rangle-\frac{140}{11}\,\langle\!\langle Q^{(3)}\rangle\!\rangle\right)\\
&\times\left\{u^3(6u^2-15u+10)\ln u+\bar u^3(6\bar u^2-15\bar u+10)\ln\bar u-\frac{1}{8}\,u\bar u\left(13\xi^2-21\right)\right\}.
\end{aligned} \tag{4.25}$$

Both (4.24) and (4.25) agree, for the ρ meson, with the results obtained in ref. [16]. The numerics of the above DAs will be discussed in the next section.

5. Models for distribution amplitudes

In this section we compile the numerical estimates of all necessary parameters and present explicit models of the twist-4 two-particle DAs introduced in sections 3 and 4. The important point is that these DAs are related to three-particle ones by exact QCD EOM and have to be used together: this guarantees the consistency of the approximation. Our model thus introduces a minimum number of non-perturbative parameters, which are defined as matrix elements of certain local operators between the vacuum and the meson state, and which we estimate using QCD sum rules. More sophisticated models can be constructed in a systematic way by adding contributions of higher conformal partial waves when estimates of the relevant non-perturbative matrix elements will become available.

Our approach involves the implicit assumption that the conformal partial wave expansion is well convergent. This can be justified rigorously at large scales, since the anomalous dimensions of all involved operators increase logarithmically with the conformal spin j, but is non-trivial at relatively low scales of order $\mu \sim (1\text{–}2)\,\mathrm{GeV}$ which we choose as reference scale. An upper bound for the contribution of higher partial waves can be obtained from the renormalon model.

Since orthogonal polynomials of high orders are rapidly oscillating functions, a truncated expansion in conformal partial waves is, almost necessarily, oscillatory as well. Such a behaviour is clearly unphysical, but this does not constitute a real problem since physical observables are given by convolution integrals of DAs with smooth coefficient functions. A classical example for this feature is the $\gamma\gamma^*$-meson form factor, which is governed by the quantity

$$\int du\, \frac{1}{u}\, \phi(u) \sim \sum a_i,$$

where the coefficients a_i are exactly the "reduced matrix elements" in the conformal expansion. The oscillating terms are averaged over and strongly suppressed. Stated otherwise: models of DAs should generally be understood as distributions (in the mathematical sense).

All relevant numerical input parameters for our model DAs are given in tables 1 and 2, at the scale $\mu = 1\,\text{GeV}$, which is appropriate for QCD sum-rule results, and at the scale $\mu = 2\,\text{GeV}$, using the scaling relations given in sections 3 and 4, to facilitate the comparison with future lattice determinations of these quantities.

The parameters related to twist-2 matrix elements have been determined using various methods; see the discussion in ref. [17]. Matrix elements of twist-3 operators were also discussed in ref. [17]. Twist-4 matrix elements for the ρ were estimated a long time ago from QCD sum rules [16, 23, 27]. In this paper, we perform a complete reanalysis of these parameters and also include G-parity-breaking effects relevant for the K^* and SU(3) breaking for the ϕ meson. The corresponding sum rules and plots are given in the appendices.

For the chiral-even parameter $\zeta_{4\rho}^{\parallel}$ we find $\zeta_{4\rho}^{\parallel} = 0.07 \pm 0.03$, which agrees with our older result $\zeta_{4\rho}^{\parallel} = 0.15 \pm 0.10$ [16] within errors. The change is due to updated input parameters. Another parameter, $\widetilde{\omega}_{4\rho}^{\parallel}$, was estimated, in ref. [16], from a correlation function of currents with different chirality, by dividing the leading contribution (a dimension-5 power correction) by the typical hadronic scale. The result $\widetilde{\omega}_{4\rho}^{\parallel} = 0.1 \pm 0.1$ is a crude estimate. In this paper we obtain $\widetilde{\omega}_{4\rho}^{\parallel} = -0.03 \pm 0.01$, from a careful analysis of various sum rules. This result is smaller than the previous estimate and negative, in agreement with the prediction based on the renormalon model (3.23). Th absolute size is smaller than the renormalon-model prediction, which is not significant, however, as the intrinsic renormalisation scale at which the model is valid is not fixed.

Another important result is that we find $\zeta_4^{\perp} + \widetilde{\zeta}_4^{\perp} \neq 0$. This parameter is usually set to zero, based on the observation that the leading contribution to the correlation function of the corresponding quark-quark-gluon operator with the electromagnetic current vanishes [23]. Similarly, as discussed in ref. [13], there is no leading renormalon contribution to this operator, which implies $\zeta_4^{\perp} + \widetilde{\zeta}_4^{\perp} = 0$ in the renormalon model. In appendix C we carefully investigate a number of different sum rules for $\zeta_4^{\perp} \pm \widetilde{\zeta}_4^{\perp}$, which are mutually consistent and yield the results given in table 2, with $\zeta_4^{\perp} + \widetilde{\zeta}_4^{\perp} \neq 0$. On the other hand, our result for $\zeta_4^{\perp} - \widetilde{\zeta}_4^{\perp}$ is consistent with older estimates based on the analysis of the leading contribution to the chiral-odd sum rules [23], albeit a factor two smaller.

	ρ		K^*		ϕ	
	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$
$f_V^\parallel$ [MeV]	216(3)	216(3)	220(5)	220(5)	215(5)	215(5)
$f_V^\perp$ [MeV]	165(9)	145(4)	185(9)	163(8)	186(9)	164(8)
$a_1^\parallel$	0	0	0.03(2)	0.02(2)	0	0
$a_1^\perp$	0	0	0.04(3)	0.03(3)	0	0
$a_2^\parallel$	0.15(7)	0.10(5)	0.11(9)	0.08(6)	0.18(8)	0.13(6)
$a_2^\perp$	0.14(6)	0.11(5)	0.10(8)	0.08(6)	0.14(7)	0.11(5)
$\zeta_{3V}^\parallel$	0.030(10)	0.020(9)	0.023(8)	0.015(6)	0.024(8)	0.017(6)
$\widetilde{\lambda}_{3V}^\parallel$	0	0	0.035(15)	0.017(8)	0	0
$\widetilde{\omega}_{3V}^\parallel$	$-0.09(3)$	$-0.04(2)$	$-0.07(3)$	$-0.03(2)$	$-0.045(15)$	$-0.022(8)$
$\kappa_{3V}^\parallel$	0	0	0.000(1)	$-0.001(2)$	0	0
$\omega_{3V}^\parallel$	0.15(5)	0.09(3)	0.10(4)	0.06(3)	0.09(3)	0.06(2)
$\lambda_{3V}^\parallel$	0	0	$-0.008(4)$	$-0.004(2)$	0	0
$\kappa_{3V}^\perp$	0	0	0.003(3)	$-0.001(2)$	0	0
$\omega_{3V}^\perp$	0.55(25)	0.37(19)	0.3(1)	0.2(1)	0.20(8)	0.15(7)
$\lambda_{3V}^\perp$	0	0	$-0.025(20)$	$-0.015(10)$	0	0

Table 1: Decay constants and twist-2 and -3 hadronic parameters at the scale $\mu = 1\,\mathrm{GeV}$ and scaled up to $\mu = 2\,\mathrm{GeV}$. The sign of the twist-3 parameters corresponds to the sign convention for the strong coupling defined by the covariant derivative $D_\mu = \partial_\mu - igA_\mu^a t^a$; they change sign if g is fixed by $D_\mu = \partial_\mu + igA_\mu^a t^a$. Numbers taken from ref. [17], see also ref. [8] for a detailed discussion of the decay constants.

The resulting four two-particle twist-4 DAs, as given by (3.26), (3.28), (4.24) and (4.25), are shown in figures 1 and 2. We use the renormalon-model predictions for all matrix elements which are not known from a direct calculation. For $\zeta_4^{\perp,\mathrm{R}}$, we use $\zeta_4^{\perp,\mathrm{R}} = (\zeta_4^\perp - \widetilde{\zeta}_4^\perp)/2$. The SU(3) breaking is moderate in $\phi_4^{\parallel,\perp}$, but obviously more pronounced for $\psi_4^{\parallel,\perp}$. This feature is mainly due to the different shape of the asymptotic DAs which vanish at the end-points for ϕ_4, but are non-zero for ψ_4. As is seen from the behaviour of $\psi_{4;K^*}^\parallel$ in particular, figure 1, the finite mass corrections in m_s change the shape of the DA noticeably for $u \to 1$; this result is dominated by the terms linear in m_s. Keeping all quark masses, the behaviour very close to the end-points is given by $m_q(m_s - m_q)\ln\bar{u}$ for $u \to 1$ and $m_s(m_s - m_q)\ln u$ for $u \to 0$. For $u \to 0$ the logarithmic term is dominant and causes the marked asymmetry in the dashed (green) curve in the left panel of figure 1. For the ϕ meson, the logarithms vanish as $m_q \to m_s$. A similar effect is at play for $\psi_{4K^*}^\perp$,

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	ρ		K^*		ϕ	
	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$	$\mu = 1\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$
$\zeta_4^\parallel$	0.07(3)	0.06(2)	0.02(2)	0.02(2)	0.00(2)	0.00(2)
$\widetilde{\omega}_4^\parallel$	$-0.03(1)$	$-0.02(1)$	$-0.02(1)$	$-0.01(1)$	$-0.02(1)$	$-0.01(1)$
$\zeta_4^\perp$	$-0.03(5)$	$-0.02(3)$	$-0.01(3)$	$-0.01(2)$	$-0.01(3)$	$-0.01(2)$
$\widetilde{\zeta}_4^\perp$	$-0.08(5)$	$-0.05(3)$	$-0.05(4)$	$-0.04(2)$	$-0.03(4)$	$-0.02(2)$
$\kappa_{4K^*}^\parallel$	0	0	$-0.025(5)$	$-0.020(4)$	0	0
$\kappa_{4K^*}^\perp$	0	0	0.013(5)	0.011(5)	0	0

Table 2: Twist-4 parameters at the scale $\mu = 1\,\mathrm{GeV}$ and scaled up to 2 GeV. Sign convention for the strong coupling g as for twist-3 parameters in table 1.

figure 2, but is slightly less marked numerically. Due to the dominance of these finite-mass corrections, the renormalon model alone gives only a poor description of the full DAs, see figure 3. In particular the size of the apex at $u = 1/2$ is considerably underestimated. The apex is actually dominated by the contribution of $a_0^{\parallel,\perp} = 1$ to the DAs, see eqs. (3.27) and (3.29). Technically speaking, this term is a mass correction and hence not included in the renormalon model. Despite this shortcoming, the renormalon model is very useful for estimating otherwise only poorly constrained higher-conformal waves of twist-4 DAs, and in particular G-parity-breaking parameters.

6. Summary and conclusions

In this paper we have studied the twist-4 two- and three-particle distribution amplitudes of ρ, K^* and ϕ mesons in QCD and expressed them in a model-independent way by a minimal number of non-perturbative parameters. The work presented here is an extension of refs. [15 − 17] and completes the analysis of SU(3)-breaking corrections by also including G-parity-breaking corrections in $m_s - m_q$ to twist-4 distribution amplitudes. Our main results are the expressions for twist-4 two-particle distribution amplitudes, eqs. (3.26), (3.28), (4.24), (4.25), and the complete set of twist-4 input parameters, table 2. With these results, a complete set of light-meson DAs of twist 2, 3 and 4 is available for both pseudoscalar and vector mesons.

Our approach consists of two components. One is the use of the QCD equations of motion, which allow dynamically dependent DAs to be expressed in terms of independent ones. The other ingredient is conformal expansion, which makes it possible to separate transverse and longitudinal variables in the wave functions, the former ones being governed by renormalisation-group equations, the latter ones being described in terms of irreducible representations of the corresponding symmetry group. We have derived expressions for all

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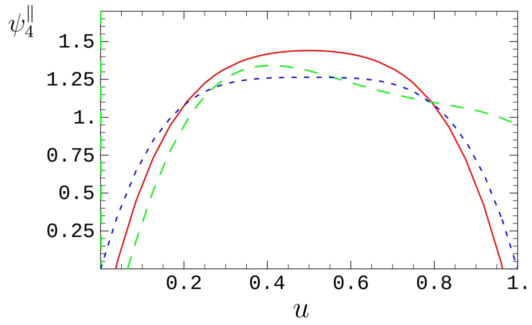

Figure 1: [Colour online] Left panel: $\psi_4^\parallel$, (3.26), as a function of u for the central value of the hadronic parameters, for $\mu = 1\,\mathrm{GeV}$. Solid [red] line: $\psi_{4;\rho}^\parallel$, dashed [green]: $\psi_{4;K^*}^\parallel$, short-dashed [blue[: $\psi_{4;\phi}^\parallel$. Right panel: same for $\phi_4^\parallel$, (3.28).

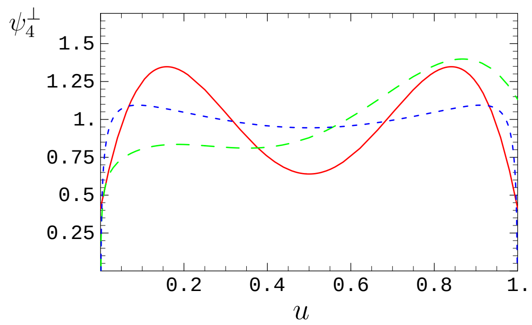

Figure 2: [Colour online] Same as figure 1 for $\psi_4^\perp$, (4.24), and $\phi_4^\perp$, (4.25).

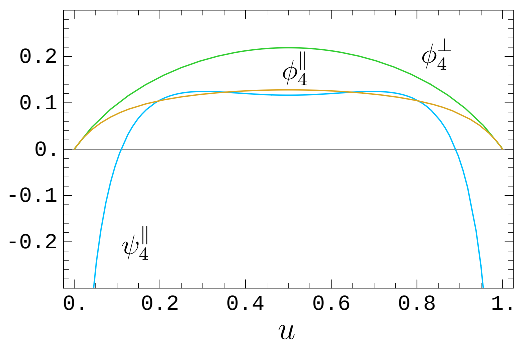

Figure 3: [Colour online] Renormalon-model predictions for $\phi_{4;\rho}^{\parallel,\perp}$ and $\psi_{4;\rho}^\parallel$; $\psi_{4;\rho}^\perp = 0$ in the renormalon model.

twist-4 two- and three-particle distribution amplitudes to NLO in the conformal expansion, including both chiral corrections $\mathcal{O}(m_s + m_q)$ and G-parity-breaking corrections $\mathcal{O}(m_s - m_q)$; the corresponding formulas are given in sections 3 and 4. We have also generalized the renormalon model of ref. [13] to describe SU(3)-breaking contributions to high-order conformal partial waves.

We have done a complete reanalysis of the numerical values of the relevant higher-twist hadronic parameters from QCD sum rules. Our sum rules can be compared, in the chiral limit, with existing calculations for the ρ [16, 27]. Whenever possible, we have aimed at determining these matrix elements from more than one sum rule; we find mutually consistent results, which provides a consistency check of the approach. Our final numerical results, at the scales 1 and 2 GeV, are collected in table 2. Any substantial improvement of these results will require input from alternative non-perturbative methods, in particular lattice calculations.

We hope that our results will contribute to a better understanding of SU(3)-breaking effects in hard exclusive processes and in particular in the decays of $B_{u,d}$ and B_s mesons into final states containing light vector mesons.

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A. Non-local operator identities

For completeness, we quote the following non-local operator identities from refs. [15, 21]:

$$\frac{\partial}{\partial x_\mu}\,\bar{q}(x)\gamma_\mu s(-x) = -i\int_{-1}^{1} dv\, v\bar{q}(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu s(-x)+i(m_s+m_q)\bar{q}(x)s(-x), \tag{A.1}$$

$$\partial_\mu\{\bar{q}(x)\gamma_\mu s(-x)\} = -i\int_{-1}^{1} dv\,\bar{q}(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu s(-x) - i(m_s-m_q)\bar{q}(x)s(-x), \tag{A.2}$$

$$\begin{aligned}\partial_\mu\bar{q}(x)\sigma_{\mu\nu}s(-x) &= -i\,\frac{\partial}{\partial x_\nu}\,\bar{q}(x)s(-x) + \int_{-1}^{1} dv\, v\bar{q}(x)x_\rho g G_{\rho\nu}(vx)s(-x)\\ &\quad - i\int_{-1}^{1} dv\,\bar{q}(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}s(-x)-(m_s+m_q)\bar{q}(x)\gamma_\nu s(-x),\end{aligned} \tag{A.3}$$

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\,\bar{q}(x)\sigma_{\mu\nu}s(-x) &= -i\partial_\nu\bar{q}(x)s(-x) + \int_{-1}^{1} dv\,\bar{q}(x)x_\rho g G_{\rho\nu}(vx)s(-x)\\ &\quad - i\int_{-1}^{1} dv\, v\bar{q}(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}s(-x)+(m_s-m_q)\bar{q}(x)\gamma_\nu s(-x).\end{aligned} \tag{A.4}$$

Here ∂_μ is the total derivative defined as

$$\partial_\mu\left\{\bar{q}(x)\Gamma s(-x)\right\} \equiv \frac{\partial}{\partial y_\mu}\left\{\bar{q}(x+y)[x+y,-x+y]\Gamma s(-x+y)\right\}\Big|_{y\to 0}.$$

By taking matrix elements of the above relations between the vacuum and the meson state, one obtains exact integral representations for those DAs that are not dynamically independent.

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B. Chiral-even twist-4 parameters

In this appendix we calculate the parameters $\zeta_{4V}^{\parallel}$ and $\widetilde{\omega}_{4V}^{\parallel}$ defined in eqs. (3.12) and (3.19). We shall obtain them from QCD sum rules, using various correlation functions of either identical currents, so-called *diagonal* sum rules, or different currents, so-called *non-diagonal* sum rules. We shall further distinguish between *pure-parity* (PP) and *mixed-parity* (MP) sum rules, depending on the parity of hadronic states that contribute to these correlation functions.

Let us first consider the non-diagonal correlation function

$$z^\mu z^\nu i \int d^4y e^{-ipy}\langle 0|T\bar{q}(z)g\widetilde{G}_{\mu\alpha}(vz)\gamma^\alpha\gamma_5 s(0)\bar{s}(y)\gamma_\nu q(y)|0\rangle = (pz)^2\int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2+v\alpha_3)}\pi(\underline{\alpha}), \tag{B.1}$$

where both currents have the same chirality. The integration measure $\mathcal{D}\underline{\alpha}$ is defined in (2.17). This is a MP correlation function, with both $J^P = 1^-$ and 0^+ states contributing. We have calculated the OPE including condensates up to dimension 6:

$$\begin{aligned}
\pi(\underline{\alpha}) = & -\frac{\alpha_s}{2\pi^3}p^2\ln\frac{\mu^2}{-p^2}\alpha_1\alpha_2\alpha_3\left\{\frac{1}{\alpha_1(1-\alpha_1)}+\frac{1}{\alpha_2(1-\alpha_2)}\right\} - \frac{1}{6p^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\delta(\alpha_3)\\
& +\frac{1}{3p^2}\frac{\alpha_s}{\pi}m_q\langle\bar{q}q\rangle\Big[\bar{\alpha}_3(\alpha_3-3)\delta(\alpha_2)-\alpha_3\bar{\alpha}_3\delta'(\alpha_2)\Big]\\
& +\frac{1}{3p^2}\frac{\alpha_s}{\pi}m_s\langle\bar{s}s\rangle\Big[\bar{\alpha}_3(\alpha_3-3)\delta(\alpha_1)-\alpha_3\bar{\alpha}_3\delta'(\alpha_1)\Big]\\
& -\frac{2}{3p^2}\frac{\alpha_s}{\pi}\left[\alpha_3^2\ln\frac{\mu^2}{-p^2}-\alpha_3^2\ln(\alpha_3\bar{\alpha}_3)+\bar{\alpha}_3\alpha_3\right]\Big[m_s\langle\bar{q}q\rangle\delta(\alpha_2)+m_q\langle\bar{s}s\rangle\delta(\alpha_1)\Big]\\
& +\frac{1}{p^4}\Big[\frac{8}{27}\pi\alpha_s\langle\bar{s}s\rangle^2+\frac{1}{3}m_s\langle\bar{s}\sigma gGs\rangle\Big]\delta(\alpha_1)\delta(\alpha_3)\\
& +\frac{1}{p^4}\Big[\frac{8}{27}\pi\alpha_s\langle\bar{q}q\rangle^2+\frac{1}{3}m_q\langle\bar{q}\sigma gGq\rangle\Big]\delta(\alpha_2)\delta(\alpha_3)\\
& -\frac{16}{9p^4}\pi\alpha_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\delta(\alpha_1)\delta(\alpha_2)\,.
\end{aligned}\tag{B.2}$$

In the local limit and zero quark masses, the result agrees with the calculation in ref. [27]. In the product of δ-functions $\delta'(\alpha_{1,2})\delta(\alpha_1+\alpha_2+\alpha_3-1)$, δ has to be integrated over before δ'.

From (B.2) we obtain the following sum rules:

$$\begin{aligned}
(f_{K^*}^{\parallel})^2 m_{K^*}^2\zeta_{4K^*}^{\parallel}e^{-m_{K^*}^2/M^2} = & -\frac{\alpha_s}{18\pi^3}M^4\left\{1-\Gamma(2,s_0/M^2)\right\}+\frac{4}{9}\frac{\alpha_s}{\pi}\left(m_q\langle\bar{q}q\rangle+m_s\langle\bar{s}s\rangle\right)\\
& +\frac{2}{9}\frac{\alpha_s}{\pi}\left(m_s\langle\bar{q}q\rangle+m_q\langle\bar{s}s\rangle\right)\left\{\frac{8}{3}+\gamma_E-\ln\frac{M^2}{\mu^2}+\Gamma(0,s_0/M^2)\right\}\\
& +\frac{1}{6}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{1}{3M^2}\left(m_q\langle\bar{q}gG\sigma q\rangle+m_s\langle\bar{s}gG\sigma s\rangle\right)\\
& +\frac{8\pi\alpha_s}{27M^2}\left\{\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right\}-\frac{16\pi\alpha_s}{9M^2}\langle\bar{q}q\rangle\langle\bar{s}s\rangle\,,
\end{aligned}\tag{B.3}$$

$\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3\,\mathrm{GeV}^3$	$\langle \bar{s}s \rangle = (1-\delta_3)\,\langle \bar{q}q \rangle$
$\langle \bar{q}\sigma gGq \rangle = m_0^2\,\langle \bar{q}q \rangle$	$\langle \bar{s}\sigma gGs \rangle = (1-\delta_5)\langle \bar{q}\sigma gGq \rangle$
$\left\langle \frac{\alpha_s}{\pi}\,G^2 \right\rangle = (0.012 \pm 0.006)\,\mathrm{GeV}^4$	$\langle g^3 f G^3 \rangle = (0.08 \pm 0.02)\,\mathrm{GeV}^6$ [28]
$m_0^2 = (0.8 \pm 0.1)\,\mathrm{GeV}^2, \quad \delta_3 = 0.2 \pm 0.2, \quad \delta_5 = 0.2 \pm 0.2$	
$\overline{m}_s(2\,\mathrm{GeV}) = (100 \pm 20)\,\mathrm{MeV} \quad \longleftrightarrow \quad \overline{m}_s(1\,\mathrm{GeV}) = (133 \pm 27)\,\mathrm{MeV}$	
$\alpha_s(m_Z) = 0.1176 \pm 0.002 \;\longleftrightarrow\; \alpha_s(1\,\mathrm{GeV}) = 0.497 \pm 0.005$	

Table 3: Input parameters for sum rules at the renormalization scale $\mu = 1\,\mathrm{GeV}$. The value of m_s is obtained from unquenched lattice calculations with $N_f = 2$ light quark flavours as summarized in ref. [29], which agrees with the results from QCD sum-rule calculations [30]. $\alpha_s(m_Z)$ is the PDG average [31].

$$\begin{aligned}
(f_{K^*}^{\parallel})^2 m_{K^*}^2 \widetilde{\omega}_{4K^*}^{\parallel} e^{-m_{K^*}^2/M^2} &= \frac{5\alpha_s}{2592\pi^3}\,M^4\left\{1-\Gamma(2,s_0/M^2)\right\} - \frac{19}{648}\,\frac{\alpha_s}{\pi}\,(m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle)\\
&\quad+\frac{11}{324}\,\frac{\alpha_s}{\pi}\,(m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle)\left\{\frac{8}{3}+\gamma_E-\ln\frac{M^2}{\mu^2}+\Gamma(0,s_0/M^2)\right\}\\
&\quad-\frac{1}{27}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle - \frac{2}{27M^2}\,(m_q\langle\bar{q}gG\sigma q\rangle + m_s\langle\bar{s}gG\sigma s\rangle)\\
&\quad-\frac{16\pi\alpha_s}{243M^2}\left\{\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right\} - \frac{40\pi\alpha_s}{81M^2}\,\langle\bar{q}q\rangle\langle\bar{s}s\rangle\,,
\end{aligned}\tag{B.4}$$

where

$$\Gamma(a, s_0/M^2) = \frac{1}{(M^2)^a}\int_{s_0}^{\infty} ds s^{a-1} e^{-s/M^2}.$$

(B.3) follows from (B.2) by integration over $\mathcal{D}\underline{\alpha}$ with the weight factor 1, and (B.4) by integration with weight factor $(\alpha_3 - 4/9)/2$, see eq. (3.19). The sum rules for ρ are obtained by letting $s \to q$ and those for ϕ by letting $q \to s$.

We evaluate the above sum rules using the input parameters collected in tables 1 and 3; the results, for central values of the input parameters, are shown in figure 4.

The sum rules are dominated by the contribution of the gluon condensate; the impact of the specific value of the continuum threshold s_0 is only moderate. The figure also shows that the impact of SU(3)-breaking is very relevant: the contributions of the quark and mixed condensate reduce the values of $\zeta_{4V}^{\parallel}$ and $\widetilde{\omega}_{4V}^{\parallel}$. In the Borel-window $1\,\mathrm{GeV}^2 < M^2 < 2\,\mathrm{GeV}^2$, and including the input-parameter uncertainties given in table 3, we find, at the scale $\mu = 1\,\mathrm{GeV}$:

$$\begin{aligned}
\zeta_{4\rho}^{\parallel} &= 0.07 \pm 0.03\,, & \widetilde{\omega}_{4\rho}^{\parallel} &= -0.03 \pm 0.01\,,\\
\zeta_{4K^*}^{\parallel} &= 0.02 \pm 0.02\,, & \widetilde{\omega}_{4K^*}^{\parallel} &= -0.02 \pm 0.01\,,\\
\zeta_{4\phi}^{\parallel} &= 0.00 \pm 0.02\,, & \widetilde{\omega}_{4\phi}^{\parallel} &= -0.02 \pm 0.01\,.
\end{aligned}\tag{B.5}$$

We have added in quadrature all individual sources of uncertainty. The total error is dominated by that of the gluon condensate.

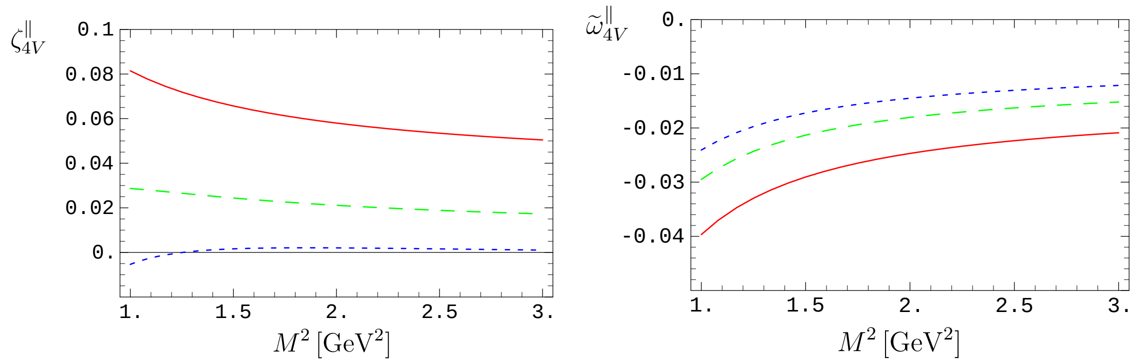

Figure 4: [Colour online] $\zeta^{\parallel}_{4V}$ (left) and $\widetilde{\omega}^{\parallel}_{4V}$ (right) from the non-diagonal MP sum rules (B.3) and (B.4) as functions of M^2, for central values of the input parameters. Solid [red] lines: ρ ($s_0 = 1.5\,\mathrm{GeV}^2$), long dashes [green]: K^* ($s_0 = 1.8\,\mathrm{GeV}^2$), short dashes [blue]: ϕ ($s_0 = 2\,\mathrm{GeV}^2$). All parameters are evaluated at the scale $\mu = 1\,\mathrm{GeV}$.

For $\zeta^{\parallel}_{4K^*}$, we also consider diagonal sum rules which can be obtained from the correlation function

$$\Pi^V_{\mu\nu} = i\int d^4x\, e^{ipx}\,\langle 0|T J^V_\mu(x)(J^V_\nu)^\dagger(0)|0\rangle \;=\; p_\mu p_\nu\,\Pi^V_0(p^2) - g_{\mu\nu}\,\Pi^V_1(p^2)\,, \qquad \text{(B.6)}$$

with the current $J^V_\mu = \bar q\, g\widetilde{G}_{\mu\alpha}\gamma_\alpha\gamma_5 s$. For ρ, $\Pi^V_{0,1}$ was calculated in ref. [27], while the SU(3)-corrections were calculated in ref. [14], including contributions from condensates up to dimension 8. The suitability of this correlation function for extracting $\zeta^{\parallel}_{4K^*}$ is not immediately obvious: Π^V_0 contains contributions not only of vector mesons, but also of hybrid 0^+ mesons, whose coupling to J^V_μ is much larger than that of the K^*, ruling out the possibility to construct a MP sum rule for $\zeta^{\parallel}_{4K^*}$. This situation is qualitatively different from that of the non-diagonal correlation function (B.1), where the presence of $\bar s\gamma_\nu q$ removes all contributions from hybrid mesons. We hence focus on the PP function Π^V_1. From ref. [14], we quote

$$\begin{aligned}
\Pi^V_1 \;=\;& \frac{\alpha_s}{240\pi^3}\,p^6\ln\frac{\mu^2}{-p^2} - \frac{1}{36}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle p^2\ln\frac{\mu^2}{-p^2} \\
&+ \frac{\alpha_s}{6\pi}[m_q\langle\bar qq\rangle + m_s\langle\bar ss\rangle]p^2\ln\frac{\mu^2}{-p^2} + \frac{\alpha_s}{18\pi}[m_s\langle\bar qq\rangle + m_q\langle\bar ss\rangle]p^2\ln\frac{\mu^2}{-p^2} \\
&+ \frac{8\pi\alpha_s}{9}\langle\bar qq\rangle\langle\bar ss\rangle - \frac{1}{192\pi^2}\,\langle g^3fG^3\rangle \\
&- \frac{19}{144}\frac{\alpha_s}{\pi}[m_q\langle\bar q\sigma gGq\rangle + m_s\langle\bar s\sigma gGs\rangle]\ln\frac{\mu^2}{-p^2} \\
&- \frac{19}{144}\frac{\alpha_s}{\pi}[m_s\langle\bar q\sigma gGq\rangle + m_q\langle\bar s\sigma gGs\rangle]\ln\frac{\mu^2}{-p^2} \\
&+ \frac{25\pi\alpha_s}{162p^2}m_0^2[\langle\bar qq\rangle^2 + \langle\bar ss\rangle^2] - \frac{181\pi\alpha_s}{162p^2}m_0^2\langle\bar qq\rangle\langle\bar ss\rangle \\
&+ \frac{\pi}{18p^2}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle[m_q\langle\bar qq\rangle + m_s\langle\bar ss\rangle] + \frac{\pi}{6p^2}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle[m_s\langle\bar qq\rangle + m_q\langle\bar ss\rangle]\,. \qquad \text{(B.7)}
\end{aligned}$$

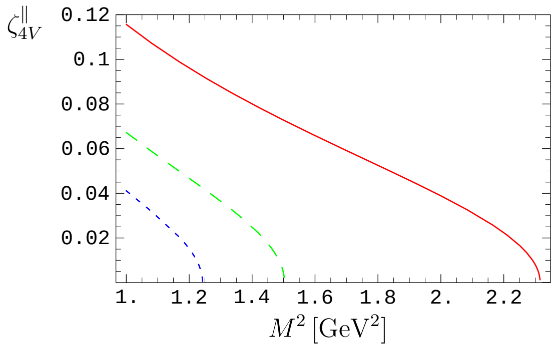

Figure 5: [Colour online] $\zeta_{4V}^{\parallel}$ from (B.8). Solid [red] line: ρ, long dashes [green]: K^*, short dashes [blue]: ϕ. Same input parameters as in figure 4.

The PP sum rule for $\zeta_{4K^*}^{\parallel}$ is

$$\left(f_K^{\parallel}\right)^2 m_{K^*}^6 (\zeta_{4K^*}^{\parallel})^2 \, e^{-m_{K^*}^2/M^2} = \mathcal{B}_{\rm sub}\Pi_1^V \,, \tag{B.8}$$

where $\mathcal{B}_{\rm sub}\Pi_1^V$ is the continuum-subtracted Borel transform of Π_1^V, which we define as

$$\mathcal{B}_{\rm sub} \int_0^\infty ds \, \frac{\rho(s)}{s-p^2} = \int_0^{s_0} ds \, e^{-s/M2} \rho(s) \,,$$

in terms of the dispersive representation of Π_1^V. For ρ, the above sum rule was derived and analysed in ref. [27]. It features a large negative contribution from the gluon condensate which, for Borel parameters $M^2 \sim 2\,\text{GeV}^2$ and continuum thresholds s_0 between 1.3 and $3\,\text{GeV}^2$, drives the right-hand side of (B.8) negative. In ref. [27] it was argued that this large negative contribution signals the presence of a larger mass scale $\sim 2\,\text{GeV}$ in the spectral density and is indicative for a breakdown of quark-hadron duality, at least if the usual simple continuum model with only one resonance, the ρ, is used. A remedy is the use of a more appropriate continuum model including higher mass states like $\rho(1450)$. This automatically increases s_0, but it also turns out that the coupling of $\rho(1450)$ to the gluonic current J_μ^V is larger than that of $\rho(770)$, which does not really help the determination of $\zeta_{4\rho}^{\parallel}$. An alternative is to analyse (B.8) for small $M^2 \approx 1\,\text{GeV}^2$, where duality still works and the suppression of higher-mass resonances is effective. Numerically, the sum rule is then dominated by the gluon and the dimension-8 condensate $\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle$. Clearly such a sum rule cannot give an accurate estimate of $\zeta_{4V}^{\parallel}$, so we only use it as a consistency check for the results obtained from (B.3) and, in particular, the large SU(3) breaking. The results from (B.8) are shown in figure 5. Note that the breakdown of duality sets in the earlier, the heavier the meson. Although it is not possible to extract precise values for $\zeta_{4V}^{\parallel}$, we see that the values are not inconsistent with the results from the non-diagonal sum rule, (B.5), and that in particular the relative hierarchy, $\zeta_{4\rho}^{\parallel} > \zeta_{4K^*}^{\parallel} > \zeta_{4\phi}^{\parallel}$, is reproduced.

Our final results are given in (B.5). A comparison with earlier determinations is given in section 5.

C. Chiral-odd twist-4 parameters

In this appendix we calculate

$$\zeta_{\pm}^{\perp} \equiv \zeta_{4K^*}^{\perp} \pm \widetilde{\zeta}_{4K^*}^{\perp} \,. \tag{C.1}$$

Like for $\zeta_{4K^*}^{\parallel}$, we consider both non-diagonal and diagonal sum rules – the former for all mesons, the latter only for ρ. We also consider PP and MP sum rules. To distinguish between the results of these sum rules, the following notation proves convenient:

$$\zeta_{\pm}^{\perp}\Big|_{\rm D(ND),PP(MP)} \,. \tag{C.2}$$

Let us start with the non-diagonal sum rules for $\zeta_{\pm}^{\perp}$, yielding $\zeta_{\pm}^{\perp}|_{\rm ND,PP(MP)}$. The relevant correlation function is

$$\begin{aligned}
\Pi^{\pm}_{\alpha\beta\mu\nu} &= i\int d^4y e^{-ipy}\langle 0|T[\bar{q}g(G_{\mu\nu} \pm i\widetilde{G}_{\mu\nu}\gamma_5)s](0)[\bar{s}\sigma_{\alpha\beta}q](y)|0\rangle \\
&= \frac{1}{p^2}\Big\{ [(p_\mu g_{\nu\alpha}p_\beta) - (\mu\leftrightarrow\nu)] - [\alpha\leftrightarrow\beta] \Big\}\Pi_V^{\pm} \\
&\quad + \frac{1}{p^2}\Big\{ [(p_\mu g_{\nu\alpha}p_\beta) - (\mu\leftrightarrow\nu)] - [\alpha\leftrightarrow\beta] + p^2(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})\Big\}\Pi_A^{\pm}.
\end{aligned} \tag{C.3}$$

The invariant functions $\Pi_V^{\pm}$ and $\Pi_A^{\pm}$ contain contributions of 1^- and 1^+ mesons, respectively, and can be separated by considering the two projections

$$\begin{aligned}
(pz)^2\Pi_1^{\pm} &\equiv z^\mu z^\alpha g^{\nu\beta}\Pi^{\pm}_{\alpha\beta\mu\nu} = -2\frac{(pz)^2}{p^2}\Big[\Pi_V^{\pm} + \Pi_A^{\pm}\Big], \\
\Pi_2^{\pm} &\equiv g^{\mu\alpha}g^{\nu\beta}\Pi^{\pm}_{\alpha\beta\mu\nu} = -6\Big[\Pi_V^{\pm} - \Pi_A^{\pm}\Big].
\end{aligned} \tag{C.4}$$

In calculating $\Pi_{1,2}^{\pm}$, we use dimensional regularization and the identity

$$G_{\mu\nu} - i\widetilde{G}_{\mu\nu}\gamma_5 = \frac{1}{4}\{\sigma_{\mu\nu},\sigma_{\rho\sigma}\}G^{\rho\sigma} \tag{C.5}$$

in order to avoid ambiguities with the definition of the epsilon-tensor and the γ_5 matrix in d dimensions. Here $\{\ldots,\ldots\}$ denotes the anti-commutator. We find

$$\begin{aligned}
\Pi_V^{-} &= -\frac{\alpha_s}{48\pi^3}p^4\ln\frac{\mu^2}{-p^2} - \frac{1}{3}\frac{\alpha_s}{\pi}\Big[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} - \frac{8\pi}{9p^2}\alpha_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle - \frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \\
&\quad + \frac{1}{12p^2}\Big[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle\Big] + \frac{1}{6p^2}\Big[m_q\langle\bar{s}\sigma gGs\rangle + m_s\langle\bar{q}\sigma gGq\rangle\Big] \\
&\quad - \frac{\alpha_s}{9\pi}\left[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\right] - \frac{\alpha_s}{6\pi}\left[m_s\langle\bar{s}s\rangle + m_q\langle\bar{q}q\rangle\right], \\
\Pi_A^{-} &= -\frac{\alpha_s}{48\pi^3}p^4\ln\frac{\mu^2}{-p^2} + \frac{1}{3}\frac{\alpha_s}{\pi}\Big[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} + \frac{8\pi}{9p^2}\alpha_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle - \frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \\
&\quad + \frac{1}{12p^2}\Big[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle\Big] - \frac{1}{6p^2}\Big[m_q\langle\bar{s}\sigma gGs\rangle + m_s\langle\bar{q}\sigma gGq\rangle\Big] \\
&\quad - \frac{\alpha_s}{9\pi}\left[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\right] - \frac{\alpha_s}{6\pi}\left[m_s\langle\bar{s}s\rangle + m_q\langle\bar{q}q\rangle\right],
\end{aligned}$$

$$\begin{aligned}
\Pi_V^+ &= -\frac{\alpha_s}{72\pi^3} p^4 \ln\frac{\mu^2}{-p^2} - \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \ln\frac{\mu^2}{-p^2} - \frac{1}{9}\frac{\alpha_s}{\pi}\Big[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} \\
&\quad + \frac{1}{9}\frac{\alpha_s}{\pi}\Big[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} - \frac{8\pi}{9p^2}\alpha_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle + \frac{8\pi}{27p^2}\alpha_s\Big[\langle\bar{s}s\rangle^2 + \langle\bar{q}q\rangle^2\Big] \\
&\quad + \frac{1}{12p^2}\Big[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle\Big] - \frac{5\alpha_s}{27\pi}\,[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle]\,, \\
\Pi_A^+ &= +\frac{\alpha_s}{72\pi^3} p^4 \ln\frac{\mu^2}{-p^2} + \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \ln\frac{\mu^2}{-p^2} - \frac{1}{9}\frac{\alpha_s}{\pi}\Big[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} \\
&\quad - \frac{1}{9}\frac{\alpha_s}{\pi}\Big[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle\Big]\ln\frac{\mu^2}{-p^2} - \frac{8\pi}{9p^2}\alpha_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle - \frac{8\pi}{27p^2}\alpha_s\Big[\langle\bar{s}s\rangle^2 + \langle\bar{q}q\rangle^2\Big] \\
&\quad - \frac{1}{12p^2}\Big[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle\Big] - \frac{5\alpha_s}{27\pi}\,[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle]\,. \qquad \text{(C.6)}
\end{aligned}$$

Again, the correlation functions for the ρ meson are obtained by $s \to q$, and those for ϕ by $q \to s$. The above functions allow one to derive the PP sum rules

$$\left(f_{K^*}^\perp\right)^2 m_{K^*}^4\, \zeta_\pm^\perp\Big|_{\mathrm{ND,PP}} e^{-m_{K^*}^2/M^2} = \mathcal{B}_{\mathrm{sub}}\,\Pi_V^\pm\,, \qquad \text{(C.7)}$$

and correspondingly for axial-vector mesons. As discussed in ref. [20], there are actually two strange 1^+ mesons, $K_1(1270)$ and $K_1(1400)$, which are usually interpreted as mixture of a ${}^3P_1$ state, the K_a, and a ${}^1P_1$ state, the K_b [32, 33]:

$$\begin{aligned}
K_1(1270) &= K_a\cos\theta_K - K_b\sin\theta_K, \\
K_1(1400) &= K_a\sin\theta_K + K_b\cos\theta_K.
\end{aligned}$$

The results of refs. [32, 33] indicate that the system is close to ideal mixing, i.e. $\theta_K \approx 45°$. To the accuracy needed here it is then sufficient to replace the two resonances by one effective one with the mass $m_{K_1} = 1.34\,\mathrm{GeV}$ [33]. We find that the sum rules for $\zeta_+^\perp$ and its axial-vector equivalent are dominated by the gluon condensate contribution, which implies

$$(f_{K_1}^\perp)^2 m_{K_1}^4\, \zeta_+^\perp(K_1)\Big|_{\mathrm{ND,PP}} \approx -(f_{K^*}^\perp)^2 m_{K^*}^4\, \zeta_+^\perp\Big|_{\mathrm{ND,PP}} \qquad \text{(C.8)}$$

with $\sim 30\%$ accuracy. For $\zeta_-^\perp$, no single contribution is dominant, but one still finds that $\zeta_-^\perp$ and $\zeta_-^\perp(K_1)$ have opposite sign. This is similar to the situation with PP sum rules for the G-odd twist-4 parameters $\kappa_{4K^*}^\perp$ and $\kappa_{4K_1}^\perp$ discussed in ref. [21], and has some impact on the relative size of continuum contributions in PP vs. MP sum rules. We will come back to this point below. In figure 6, we show the results for $\zeta_\pm^\perp|_{\mathrm{ND,PP}}$.

The values of s_0 are chosen in such a way as to ensure maximum stability of $\zeta_+^\perp|_{\mathrm{ND,PP}}$ in the Borel parameter. Note that the sum rules (C.7) are quite sensitive to the value of the continuum threshold, which we vary by $\pm 0.3\,\mathrm{GeV}^2$. Including this uncertainty and the variation in M^2, in the interval $1\,\mathrm{GeV}^2 < M^2 < 2\,\mathrm{GeV}^2$, and the error induced by the hadronic input parameters, tables 2 and 3, we find, at the scale $\mu = 1\,\mathrm{GeV}$:

$$\begin{aligned}
\zeta_+^\perp(\rho)\Big|_{\mathrm{ND,PP}} &= -0.12 \pm 0.04, & \zeta_-^\perp(\rho)\Big|_{\mathrm{ND,PP}} &= 0.03 \pm 0.02\,, \\
\zeta_+^\perp(K^*)\Big|_{\mathrm{ND,PP}} &= -0.07 \pm 0.02, & \zeta_-^\perp(K^*)\Big|_{\mathrm{ND,PP}} &= 0.04 \pm 0.02\,, \\
\zeta_+^\perp(\phi)\Big|_{\mathrm{ND,PP}} &= -0.05 \pm 0.02, & \zeta_-^\perp(\phi)\Big|_{\mathrm{ND,PP}} &= 0.04 \pm 0.02\,. \qquad \text{(C.9)}
\end{aligned}$$

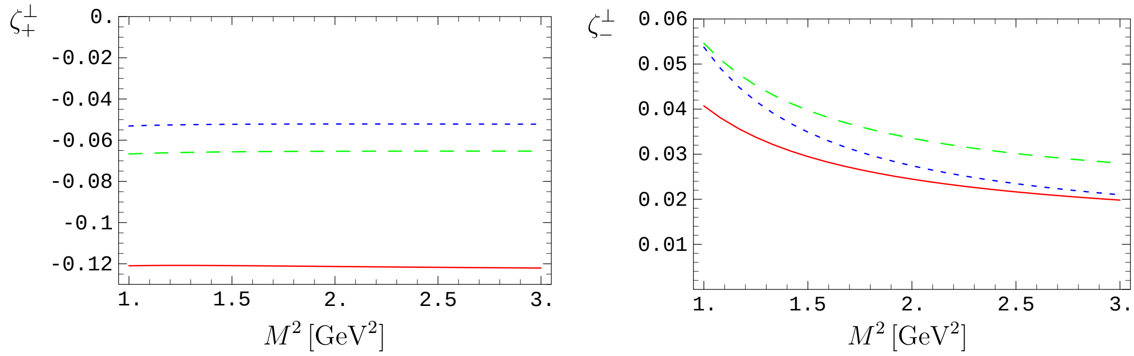

Figure 6: [Colour online] $\zeta_+^\perp|_{\rm ND,PP}$ (left) and $\zeta_-^\perp|_{\rm ND,PP}$ (right) from the PP sum rule (C.7), as functions of M^2, for $\mu = 1\,{\rm GeV}$ and central values of input parameters. Solid [red] lines: ρ ($s_0 = 1.2\,{\rm GeV}^2$), long dashes [green]: K^* ($s_0 = 1.4\,{\rm GeV}^2$), short dashes [blue]: ϕ ($s_0 = 1.8\,{\rm GeV}^2$).

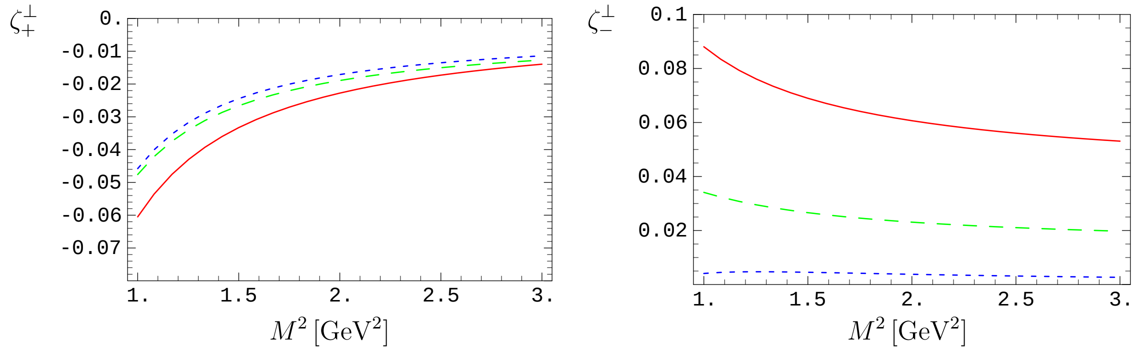

Figure 7: [Colour online] $\zeta_+^\perp|_{\rm ND,MP}$ (left) and $\zeta_-^\perp|_{\rm ND,MP}$ (right) from the MP sum rule (C.10), as functions of M^2, for $\mu = 1\,{\rm GeV}$ and central values of input parameters. Solid [red] lines: ρ ($s_0 = 1.0\,{\rm GeV}^2$), long dashes [green]: K^* ($s_0 = 1.2\,{\rm GeV}^2$), short dashes [blue]: ϕ ($s_0 = 1.5\,{\rm GeV}^2$).

We have added all individual uncertainties in quadrature. The bulk of SU(3) breaking in these quantities is due to the factor $m_V^4(f_V^\perp)^2$ in (C.7), with $m_{K^*}^4(f_{K^*}^\perp)^2/(m_\rho^4(f_\rho^\perp)^2) = 2.2$ and $m_\phi^4(f_\phi^\perp)^2/(m_\rho^4(f_\rho^\perp)^2) = 3.8$, which explains the relative hierarchy $|\zeta_+^\perp(\rho)| > |\zeta_+^\perp(K^*)| > |\zeta_+^\perp(\phi)|$. For $\zeta_-^\perp$, one has a cancellation of several terms which renders the interpretation of the hierarchy of the curves in figure 6 less clear-cut.

Let us now turn to the extraction of $\zeta_\pm^\perp$ from MP sum rules, which contain contributions from both vector and axial-vector mesons. These sum rules are derived from the combinations $(\Pi_V^\pm + \Pi_A^\pm)/p^2$ and, thanks to the factor $1/p^2$, benefit from a smaller mass dimension than the PP sum rules:

$$(f_{K^*}^\perp)^2 m_{K^*}^2\, \zeta_\pm^\perp\Big|_{\rm ND,MP}\, e^{-m_{K^*}^2/M^2} = \mathcal{B}_{\rm sub}\,\frac{1}{p^2}\left(\Pi_V^\pm + \Pi_A^\pm\right). \tag{C.10}$$

The corresponding results are shown in figure 7. The sum rule for $\zeta_+^\perp|_{\rm ND,MP}$ consists of only two terms: the quark-condensate and the four-quark-condensate contributions, with the latter dominant. Such a sum rule, sensitive to higher-dimensional condensates, is not reliable and we do not include its results into our final value for $\zeta_+^\perp$. $\zeta_-^\perp|_{\rm ND,MP}$, on the other hand, does not receive any contribution from the four-quark condensate, and is dominated by the gluon condensate. In contrast to $\zeta_\pm^\perp|_{\rm ND,PP}$, $\zeta_\pm^\perp|_{\rm ND,MP}$ is rather insensitive to the precise value of the continuum threshold s_0. The reason for this is the different sign of vector and axial-vector contributions to the hadronic side of the sum rule: as mentioned above, the 1^+ and 1^- matrix elements tend to have different sign, and hence the resonance contributions to the sum rule tend to cancel, reducing the size of the continuum contribution. We choose s_0 for the MP sum rules slightly below that for PP sum rules, to account for the lower mass of the 1^+ ground state as compared to the first 1^- excitation. Using again the hadronic input parameters from table 2 and table 3, and varying s_0 by $\pm 0.3\,\mathrm{GeV}^2$, and M^2 in the window $1\,\mathrm{GeV}^2 < M^2 < 2\,\mathrm{GeV}^2$, we obtain the following results for $\zeta_-^\perp|_{\rm ND,MP}$ (at the scale $\mu = 1\,\mathrm{GeV}$):

$$\zeta_-^\perp(\rho)\Big|_{\rm ND,MP} = 0.07 \pm 0.03\,, \quad \zeta_-^\perp(K^*)\Big|_{\rm ND,MP} = 0.03 \pm 0.02\,, \quad \zeta_-^\perp(\phi)\Big|_{\rm ND,MP} = 0 \pm 0.02\,. \tag{C.11}$$

We have added all individual uncertainties in quadrature. We do not give any results for $\zeta_+^\perp|_{\rm ND,MP}$ because this sum rule is dominated by higher-dimension condensates and hence not reliable. Again SU(3) breaking in the above numbers is dominated by the overall factors $m_V^2(f_V^\perp)^2$ in (C.10). Both PP and MP non-diagonal sum rules agree about the signs of $\zeta_\pm^\perp$: $\zeta_-^\perp > 0$ and $\zeta_+^\perp < 0$.

Let us now discuss the diagonal sum rules for $\zeta_\pm^\perp$ which can be derived from the correlation function of two quark-antiquark-gluon currents:

$$\begin{aligned}
\Pi^{\pm\pm}_{\alpha\beta\mu\nu} &= i\int d^4y e^{-ipy}\langle 0|T\{[\bar q g(G_{\mu\nu} \pm i\widetilde G_{\mu\nu}\gamma_5)s](0)[\bar s g(G_{\alpha\beta} \pm i\widetilde G_{\alpha\beta}\gamma_5)q](y)\}|0\rangle \\
&= \frac{1}{p^2}\Big\{ [(p_\mu g_{\nu\alpha}p_\beta) - (\mu\leftrightarrow\nu)] - [\alpha\leftrightarrow\beta]\Big\}\Pi_V^{\pm\pm} \\
&\quad + \frac{1}{p^2}\Big\{ [(p_\mu g_{\nu\alpha}p_\beta) - (\mu\leftrightarrow\nu)] - [\alpha\leftrightarrow\beta] + p^2(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})\Big\}\Pi_A^{\pm\pm}.
\end{aligned} \tag{C.12}$$

Like for the non-diagonal correlation function (C.3), the invariant functions $\Pi_V^{\pm\pm}$ and $\Pi_A^{\pm\pm}$ contain contributions of $J^P = 1^-$ and 1^+ states, respectively, and can be separated by the projections

$$\begin{aligned}
(pz)^2\Pi_1^{\pm\pm} &\equiv z^\mu z^\alpha g^{\nu\beta}\Pi^{\pm\pm}_{\alpha\beta\mu\nu} = -2\frac{(pz)^2}{p^2}\Big[\Pi_V^{\pm\pm} + \Pi_A^{\pm\pm}\Big], \\
\Pi_2^{\pm\pm} &\equiv g^{\mu\alpha}g^{\nu\beta}\Pi^{\pm\pm}_{\alpha\beta\mu\nu} = -6\Big[\Pi_V^{\pm\pm} - \Pi_A^{\pm\pm}\Big].
\end{aligned} \tag{C.13}$$

Obviously $\Pi^{+-}_{V(A)} = \Pi^{-+}_{V(A)}$. We calculate these invariant functions neglecting mass corrections and find

$$\begin{aligned}
\Pi_V^{++} &= -\frac{\alpha_s}{480\pi^3}p^6\ln\frac{-p^2}{\mu^2} + \frac{\langle g^3 fG^3\rangle}{48\pi^2} - \frac{g^2}{324}\langle\bar qq\rangle\langle\bar q\sigma gGq\rangle\frac{1}{p^2}\,, \\
\Pi_A^{++} &= -\frac{\alpha_s}{480\pi^3}p^6\ln\frac{-p^2}{\mu^2} - \frac{\langle g^3 fG^3\rangle}{48\pi^2} - \frac{g^2}{324}\langle\bar qq\rangle\langle\bar q\sigma gGq\rangle\frac{1}{p^2}\,, \\
\Pi_V^{+-} &= +\frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle p^2\ln\frac{-p^2}{\mu^2} + \frac{4\pi}{9}\alpha_s\langle\bar qq\rangle^2 + \frac{\langle g^3 fG^3\rangle}{24\pi^2}\left[\ln\frac{\mu^2}{-p^2} - \frac{1}{2}\right] + \frac{23g^2}{216}\langle\bar qq\rangle\langle\bar q\sigma gGq\rangle\frac{1}{p^2}\,,
\end{aligned}$$

$$\Pi_A^{+-} = -\frac{1}{24}\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle p^2 \ln\frac{-p^2}{\mu^2} + \frac{4\pi}{9}\alpha_s\langle\bar{q}q\rangle^2 - \frac{\langle g^3 fG^3\rangle}{24\pi^2}\left[\ln\frac{\mu^2}{-p^2} - \frac{1}{2}\right] + \frac{5g^2}{216}\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle\frac{1}{p^2}\,,$$
$$\Pi_V^{--} = -\frac{\alpha_s}{480\pi^3}p^6\ln\frac{-p^2}{\mu^2} + \frac{\langle g^3 fG^3\rangle}{48\pi^2} - \frac{29g^2}{324}\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle\frac{1}{p^2}\,,$$
$$\Pi_A^{--} = -\frac{\alpha_s}{480\pi^3}p^6\ln\frac{-p^2}{\mu^2} - \frac{\langle g^3 fG^3\rangle}{48\pi^2} + \frac{13g^2}{324}\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle\frac{1}{p^2}\,. \qquad \text{(C.14)}$$

Without quark-mass corrections, the above results only allow the calculation of the ρ couplings. We construct PP and MP sum rules for $(\zeta_+^\perp)^2$ and $(\zeta_-^\perp)^2$, and also for the product $\zeta_+^\perp\zeta_-^\perp$:

$$(f_\rho^\perp)^2 m_\rho^6\,(\zeta_\pm^\perp)^2\Big|_{\text{D,PP}} = \mathcal{B}_{\text{sub}}\Pi_V^{\pm\pm}\,, \qquad \text{(C.15)}$$
$$(f_\rho^\perp)^2 m_\rho^6\,(\zeta_+^\perp\zeta_-^\perp)\Big|_{\text{D,PP}} = \mathcal{B}_{\text{sub}}\Pi_V^{+-}\,, \qquad \text{(C.16)}$$
$$(f_\rho^\perp)^2 m_\rho^4\,(\zeta_\pm^\perp)^2\Big|_{\text{D,MP}} = \mathcal{B}_{\text{sub}}\frac{1}{p^2}\left(\Pi_V^{\pm\pm} + \Pi_A^{\pm\pm}\right), \qquad \text{(C.17)}$$
$$(f_\rho^\perp)^2 m_\rho^4\,(\zeta_+^\perp\zeta_-^\perp)\Big|_{\text{D,MP}} = \mathcal{B}_{\text{sub}}\frac{1}{p^2}\left(\Pi_V^{+-} + \Pi_A^{+-}\right). \qquad \text{(C.18)}$$

The results are shown in figures 8 and 9.

As figure 8 indicates, both (C.16) and (C.18) predict different sign for $\zeta_+^\perp$ and $\zeta_-^\perp$, which agrees with the result from non-diagonal sum rules, figures 6 and 7. In figure 9 we plot the results from (C.15) and (C.17). All four sum rules receive contributions only from perturbation theory and the dimension 8 condensate $\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle$. For both sum rules for $\zeta_+^\perp$, perturbation theory is dominant, whereas for those for $\zeta_-^\perp$, the condensate is dominant. For $\zeta_-^\perp|_{\text{D,MP}}$ it comes with a negative sign, which explains the fact that the dashed curve in figure 9 (right) only starts at large M^2: below that, the result is imaginary. A possible explanation is the contribution of hybrid $1^\pm$ states to the sum rules, both for $\zeta_-^\perp|_{\text{D,MP}}$ and $\zeta_-^\perp|_{\text{D,PP}}$. Indeed, replacing the parametrisation on the left-hand side of (C.17) by a contribution of such a state, with a mass ~ 1.5 or $2\,\text{GeV}$, and correspondingly larger $s_0 \approx 4\,\text{GeV}^2$, the coupling becomes real. In any case, the dominance of the dimension-8 term in $\langle\bar{q}q\rangle\langle\bar{q}\sigma gGq\rangle$ in these sum rules renders them unreliable and hence we discard their results. As for $\zeta_+^\perp|_{\text{D}}$, the sum rule is very dependent on s_0, which totally dominates the error budget. This is another manifestation of the fact that the currents in (C.12) have a strong coupling to hybrid states. We finally find

$$\zeta_+^\perp\Big|_{\text{D,PP}} = -0.06 \pm 0.02, \qquad \zeta_+^\perp\Big|_{\text{D,MP}} = -0.04 \pm 0.02\,. \qquad \text{(C.19)}$$

Our last task is to give final results for $\zeta_\pm^\perp$ and, equivalently, $\zeta_{4V}^\perp$ and $\widetilde{\zeta}_{4V}^\perp$, according to (C.1). Let us first discuss the ρ parameters, as only for those we have information from both diagonal and non-diagonal sum rules. For $\zeta_+^\perp$, we have three results, $\zeta_+^\perp|_{\text{D,PP}}$ and $\zeta_+^\perp|_{\text{D,MP}}$ from (C.19), dominated by perturbation theory, and $\zeta_+^\perp|_{\text{ND,PP}} = -0.12 \pm 0.04$, dominated by the gluon condensate contribution. As none of these results is a priori "better" than the others, we average over all of them to obtain our final result

$$\zeta_+^\perp(\rho, 1\,\text{GeV}) = -0.10 \pm 0.06\,. \qquad \text{(C.20)}$$

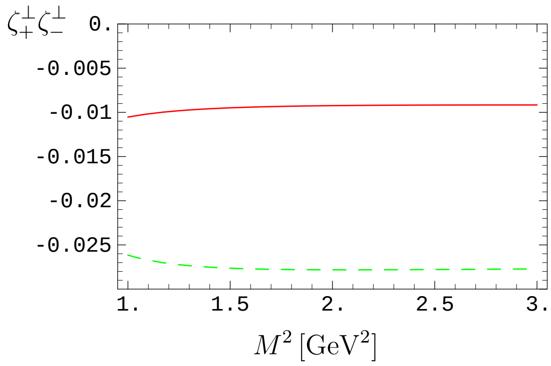

Figure 8: [Colour online] $(\zeta_+^\perp\zeta_-^\perp)_D$ for ρ from the PP sum rule (C.16) (solid [red] curve) and the MP sum rule (C.18) (dashed [green] curve).

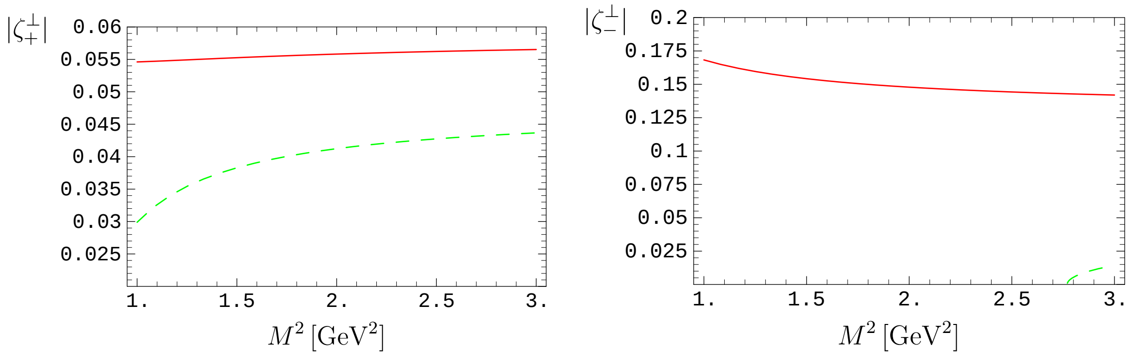

Figure 9: [Colour online] $|\zeta_+^\perp|_{\rm D}$ (left) and $|\zeta_-^\perp|_{\rm D}$ (right), for the ρ, from the sum rules (C.15) and (C.17). Solid [red] curves: PP ($s_0 = 1.2\,\mathrm{GeV}^2$), dashed [green] curves: MP sum rules ($s_0 = 1.0\,\mathrm{GeV}^2$).

The average is smaller than the MP sum result alone, which we take into account when arriving at our final results for K^* and ϕ from (C.9):

$$\zeta_+^\perp(K^*, 1\,\mathrm{GeV}) = -0.06 \pm 0.03\,, \qquad \zeta_+^\perp(\phi, 1\,\mathrm{GeV}) = -0.04 \pm 0.03\,. \tag{C.21}$$

As for $\zeta_-^\perp$, the diagonal sum rules have to be discarded, whereas the non-diagonal ones yield $\zeta_-^\perp|_{\rm ND,PP} = 0.03\pm0.02$, with the most relevant contributions from perturbation theory and the dimension 6 condensate $\langle\bar{q}q\rangle^2$, and $\zeta_-^\perp|_{\rm ND,MP} = 0.07 \pm 0.03$, with the most relevant contributions from perturbation theory and the gluon condensate. Here we obtain our final result as a straight average of PP and MP sum rules and find, from (C.9) and (C.11):

$$\zeta_-^\perp(\rho, 1\,\mathrm{GeV}) = 0.05\pm0.05\,, \quad \zeta_-^\perp(K^*, 1\,\mathrm{GeV}) = 0.04\pm0.04\,, \qquad \zeta_-^\perp(\phi, 1\,\mathrm{GeV}) = 0.02\pm0.04\,. \tag{C.22}$$

From (C.1), we also find the final results for $\zeta_{4V}^\perp$ and $\widetilde{\zeta}_{4V}^\perp$:

$$\begin{aligned}
\zeta_{4\rho}^\perp(1\,\mathrm{GeV}) &= -0.03 \pm 0.05\,, \quad \zeta_{4K^*}^\perp(1\,\mathrm{GeV}) = -0.01 \pm 0.03\,, \quad \zeta_{4\phi}^\perp(1\,\mathrm{GeV}) = -0.01 \pm 0.03\,,\\
\widetilde{\zeta}_{4\rho}^\perp(1\,\mathrm{GeV}) &= -0.08 \pm 0.05\,, \quad \widetilde{\zeta}_{4K^*}^\perp(1\,\mathrm{GeV}) = -0.05 \pm 0.04\,, \quad \widetilde{\zeta}_{4\phi}^\perp(1\,\mathrm{GeV}) = -0.03 \pm 0.04\,.
\end{aligned} \tag{C.23}$$

A comparison of our results with those from previous calculations is given in section 5.

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